

Contents

Preface	xiii
Chapter 1. Introduction	1
1.1. Dynamic response	2
1.2. Dynamic loading	2
1.2.1. Periodic loadings	3
1.2.1.1. Harmonic loadings	3
1.2.1.2. Arbitrary periodic loadings	3
1.2.2. Non-periodic loadings	4
1.2.2.1. Impulse loadings	4
1.2.2.2. Arbitrary loadings	4
1.3. Additional considerations	4
1.4. Formulation of the equation of motion	5
1.4.1. System with one mass particle	6
1.4.1.1. Newton's second law of motion	6
1.4.1.2. D'Alembert's principle	6
1.4.1.3. Virtual work principle	7
1.4.1.4. Constraints	8
1.4.2. System with many mass particles	9
1.4.3. System with deformable bodies	10
1.5. Dynamic degrees of freedom	10
1.6. Modeling a dynamic problem	12
1.6.1. Mass concentration	12
1.6.2. Rayleigh–Ritz method	13
1.6.3. Finite element method	15
1.7. Dynamic analysis of structures	18
1.8. Dynamic testing	19
1.9. Measuring vibration levels	20
1.10. Suggested reading	23

PART 1. SINGLE DEGREE OF FREEDOM SYSTEMS	25
Chapter 2. Equation of Motion	27
2.1. Response parameters	27
2.2. Immobile support	28
2.3. Effect of gravity forces	30
2.4. Motion of the support	31
Chapter 3. Free Response	37
3.1. Characteristic equation	37
3.2. Undamped free response	38
3.3. Conservation of energy	47
3.4. Damped free response	48
3.4.1. Subcritical damping	49
3.4.2. Critical damping	52
3.4.3. Overcritical damping	54
3.5. Dissipation of energy in a system with subcritical damping	55
3.6. Coulomb damping	58
3.7. Logarithmic decrement	63
Chapter 4. Forced Response to Harmonic Loading	71
4.1. Forced response of conservative systems	72
4.1.1. Forced response to cosine force	77
4.2. Beating	78
4.3. Forced response of dissipative systems	79
4.4. Steady-state response to cosine force	86
4.5. Resonance	87
4.6. Dynamic amplification factors	89
4.7. Resonant angular frequency	91
4.8. Power absorbed in steady-state forced vibration	92
4.9. Complex frequency response	97
4.10. Nyquist plot	101
4.11. Vibration measurement instruments	105
4.11.1. Displacement sensor or vibrometer	106
4.11.2. Velocity transducer or velometer	108
4.11.3. Acceleration transducer or accelerometer	108
4.12. Vibration isolation	110
4.12.1. Vertical oscillating force	110
4.12.2. Harmonic motion of the base	112
4.13. Mass eccentricity	116

Chapter 5. Measurement of Damping	123
5.1. Free-decay method	123
5.2. Amplification method	124
5.3. Half-power bandwidth method	125
5.4. Nyquist plots	129
5.5. Energy dissipated by damping	129
5.5.1. Viscous damping	129
5.5.2. Internal material damping	132
Chapter 6. Forced Response to Periodic Loading	141
6.1. Representation of a periodic function as a Fourier series	142
6.1.1. Trigonometric form of the Fourier series	142
6.1.2. Complex or exponential form of the Fourier series	148
6.2. Fourier spectrum	149
6.3. Response to periodic loading	150
6.3.1. Trigonometric Fourier series decomposition of the load function	150
6.3.2. Exponential Fourier series decomposition of the load function	155
Chapter 7. Response to Arbitrary Loading in the Time Domain	161
7.1. Response to an impulse loading	161
7.2. Dirac impulse or delta function	163
7.3. Response to a Dirac impulse	165
7.4. Duhamel integral	165
7.5. Convolution integral	167
7.6. Numerical evaluation of the Duhamel integral	170
7.6.1. Conservative system	170
7.6.2. Dissipative system	171
7.7. Response to a step load	176
7.8. Response to a linearly increasing force	178
7.9. Response to a constant force applied slowly	179
7.10. Response to impulse loads	181
7.10.1. Sinusoidal impulse	182
7.10.2. Rectangular impulse	184
7.10.3. Triangular impulse	187
7.10.4. Symmetric triangular impulse	189
7.10.5. Shock response spectra	190
Chapter 8. Forced Response to Arbitrary Loading in Frequency Domain	195
8.1. Fourier transform	195
8.2. Relationship between the frequency response function and the impulse response function	199
8.3. Discrete Fourier transform	200

8.4. Nyquist frequency	203
8.5. Fast Fourier transform: Cooley–Tukey algorithm	205
8.6. Signal flow graph	213
8.7. Calculation of double nodes	215
8.8. Calculation of the inverse fast Fourier transform	216
Chapter 9. Direct Time Integration of Linear Systems	223
9.1. General	224
9.2. Exact numerical integration for piecewise linear loading functions	226
9.3. Central difference method	229
9.4. Newmark method	236
9.4.1. Average acceleration method	238
9.4.2. Linear acceleration method	241
9.4.3. Generalization of the Newmark’s methods	243
Chapter 10. Direct Time Integration of Nonlinear Systems	249
10.1. Incremental equation of dynamic equilibrium	249
10.2. Newmark’s methods	251
10.3. Error reduction with Newton method	255
Chapter 11. Generalized Elementary Systems	271
11.1. Rigid-body assemblies	271
11.2. Flexible system	277
11.3. Elementary generalized system	281
11.4. Rayleigh method	283
11.4.1. Elementary system	283
11.4.2. Continuous system	284
11.4.3. Selection of a displacement function	287
11.4.4. Improved Rayleigh method	294
11.4.5. Discrete system	297
Chapter 12. Response to Earthquake Excitation	307
12.1. Earthquake response in the time domain	308
12.2. Response spectrum	312
12.3. Design spectrum	319
12.4. Use of design spectra	320
12.5. Earthquake intensity	323
12.6. Fourier spectrum, relative velocity spectrum and energy	324
12.7. Response of a generalized SDOF system	328
12.8. Nonlinear response	333
12.9. Inelastic response spectrum	340

PART 2. MULTI-DEGREES OF FREEDOM SYSTEMS	347
Chapter 13. Equations of Motion	349
13.1. Simplified model of a building	350
13.2. Equation of dynamic equilibrium	352
13.3. Stiffness influence coefficients	354
13.4. Static condensation	366
13.5. Support motions	368
13.5.1. Synchronous support motion of a planar system	369
13.5.2. Structure with multiple support motions	374
13.5.3. Additional mass method	379
Chapter 14. Finite Element Method	385
14.1. Finite element method: overview	385
14.2. Global formulation using the principle of virtual works	388
14.3. Local formulation using the principle of virtual work	399
14.4. Coordinate transformations	403
14.5. Generalized displacements, strains and stresses	406
14.6. Two-node truss element	411
14.7. Beam finite element	414
14.8. Beam-column element	417
14.9. Geometric stiffness matrix	421
14.9.1. Two-node truss element	422
14.9.2. Two-node beam-column element	425
14.10. Rules for assembling element matrices	428
14.11. Properties of the stiffness matrix	432
14.12. Numerical solution	433
14.13. Post-processing	438
14.14. Convergence and compatibility	439
14.15. Isoparametric elements	440
Chapter 15. Free Response of Conservative Systems	443
15.1. Physical significance of eigenvalues and eigenvectors	444
15.2. Evaluation of vibration frequencies	446
15.3. Evaluation of mode shapes	448
15.4. Flexibility matrix formulation	452
15.5. Influence of axial forces	454
15.6. Orthogonality of mode shapes	455
15.6.1. Normalization of eigenvectors	458
15.7. Comparing prediction and measured data	459
15.8. Influence of the mass matrix	462

Chapter 16. Free Response of Non-conservative Systems	469
16.1. Proportional damping matrix	469
16.2. Superposition of modal damping matrices	474
16.3. Damping measurement by harmonic excitation	476
16.4. Non-proportional damping matrix	479
16.5. Construction of non-proportional damping matrices	481
Chapter 17. Response to Arbitrary Loading by Modal Superposition . . .	487
17.1. Normal coordinates	488
17.2. Uncoupled equations of motion	489
17.3. Modal superposition method	491
17.3.1. Calculation of the response	491
17.3.1.1. Direct numerical integration	491
17.3.1.2. Calculation of Duhamel integral	491
17.3.1.3. Fourier transform	492
17.3.2. Initial conditions	492
17.3.3. Total response	493
17.3.4. Calculation of elastic forces	493
17.4. Error due to the use of a truncated eigenvector base	496
17.5. Harmonic amplification	500
17.6. Static correction	502
17.7. Modal acceleration method	504
17.8. Summary of the modal superposition method	505
Chapter 18. Modal Superposition Response to Earthquake Excitation . . .	509
18.1. Modal superposition	509
18.2. Effective modal mass	514
18.3. Error due to the use of a truncated modal base	515
18.4. Superposition of spectral responses	520
18.5. Response of systems with multiple supports	526
Chapter 19. Properties of Eigenvalues and Eigenvectors	531
19.1. Standard symmetric eigenvalue problem	531
19.2. Similarity transformations	534
19.3. Some properties of the symmetric eigenvalue problem	535
19.4. Generalized symmetric eigenvalue problem	537
19.4.1. Fundamental properties	538
19.4.2. Multiplicity of eigenvalues	541
19.5. Standard eigenvalue problem for a real non-symmetric matrix	543
19.6. Spectral shift	546
19.7. Zero masses	548
19.8. Transformation of generalized eigenvalue problems to standard form	549

19.9. Rayleigh quotient	552
19.9.1. Homogeneity property	553
19.9.2. Stationarity property	553
19.9.3. Bounding property	554
19.10. Max-min and min-max characterization of the eigenvalues	556
19.11. Cauchy's interlace theorem	559
19.12. Properties of characteristic polynomials	561
19.13. Sylvester's law of inertia	564
Chapter 20. Reduction of Coordinates	569
20.1. Kinematic constraints	570
20.2. Static condensation	574
20.3. Rayleigh analysis	579
20.4. Rayleigh–Ritz analysis	581
20.5. Load-dependent Ritz vectors	588
20.6. Guyan–Irons reduction method	600
Chapter 21. Numerical Methods for Eigenproblems	605
21.1. Iterative methods	606
21.1.1. Inverse iteration	606
21.1.2. Direct iteration	613
21.1.3. Inverse iteration with spectral shift	618
21.1.4. Inverse iteration with orthogonal deflation	620
21.2. Rotation and reflection	621
21.3. Transformation methods	623
21.3.1. Jacobi method	624
21.3.2. Generalized Jacobi method	629
21.3.3. QR iteration	637
21.4. HQRI iterations	649
21.5. Subspace iterations	658
21.5.1. Algorithm	658
21.5.2. Choice of the starting iteration vectors	661
Chapter 22. Direct Time Integration of Linear Systems	671
22.1. Multi-step methods	672
22.1.1. Multi-step methods for first-order equations	672
22.1.2. Multi-step methods to solve second-order equations	673
22.2. The central difference method	674
22.3. Houbolt method	679
22.4. Newmark methods	681
22.5. Wilson- θ method	685
22.6. Collocation methods	687

22.7. HHT- α method	692
22.8. Estimation of the highest eigenvalue	697
22.9. Stability analysis	703
22.9.1. Exact solutions	704
22.9.2. Discrete approximation	705
22.9.3. Central difference method	705
22.9.4. Houbolt method	706
22.9.5. Newmark method	707
22.9.6. Wilson- θ method	708
22.9.7. HHT- α method	709
22.10. Stability conditions	711
22.10.1. Central difference method	717
22.10.2. Newmark methods	719
22.10.3. Wilson- θ method	724
22.10.4. HHT- α method	724
22.10.5. Comparison of the various methods	725
22.11. Analysis of the consistency of a finite-difference scheme	726
22.12. Analysis of the accuracy	728
22.12.1. Accuracy of the Newmark method	729
22.12.2. Measure of the accuracy of integration schemes	730
22.13. Filtering of unwanted artificial modes and overestimation of the response	733
22.14. Selection of a numerical direct integration method	738
Chapter 23. Direct Time Integration of Nonlinear Systems	741
23.1. Incremental equation of motion	741
23.2. The central difference explicit method	742
23.3. Implicit Newmark methods	745
23.4. Error reduction with the Newton method	746
23.5. Nonlinear analysis of a building under seismic loading	751
Appendix A: Complex Numbers	757
A.1. Algebraic representation	757
A.2. Operations	758
A.3. Geometric representation	758
A.4. Trigonometric form	759
A.5. Roots	762
Bibliography	765
Index	773