

# Contents

---

<b>Preface</b>	<b>v</b>
<b>1</b>	
<b>Introduction</b>	<b>1</b>
1.1 Strategy of Experimentation	1
1.2 Some Typical Applications of Experimental Design	8
1.3 Basic Principles	11
1.4 Guidelines for Designing Experiments	14
1.5 A Brief History of Statistical Design	21
1.6 Summary: Using Statistical Techniques in Experimentation	22
1.7 Problems	23
<b>2</b>	
<b>Simple Comparative Experiments</b>	<b>25</b>
2.1 Introduction	25
2.2 Basic Statistical Concepts	27
2.3 Sampling and Sampling Distributions	30
2.4 Inferences About the Differences in Means, Randomized Designs	36
2.4.1 Hypothesis Testing	36
2.4.2 Confidence Intervals	43
2.4.3 Choice of Sample Size	44
2.4.4 The Case Where $\sigma_1^2 \neq \sigma_2^2$	48
2.4.5 The Case Where $\sigma_1^2$ and $\sigma_2^2$ Are Known	50
2.4.6 Comparing a Single Mean to a Specified Value	50
2.4.7 Summary	51
2.5 Inferences About the Differences in Means, Paired Comparison Designs	53
2.5.1 The Paired Comparison Problem	53
2.5.2 Advantages of the Paired Comparison Design	56
2.6 Inferences About the Variances of Normal Distributions	57
2.7 Problems	59

**3****Experiments with a Single Factor:  
The Analysis of Variance****65**

3.1	An Example	66
3.2	The Analysis of Variance	68
3.3	Analysis of the Fixed Effects Model	70
3.3.1	Decomposition of the Total Sum of Squares	71
3.3.2	Statistical Analysis	73
3.3.3	Estimation of the Model Parameters	78
3.3.4	Unbalanced Data	79
3.4	Model Adequacy Checking	80
3.4.1	The Normality Assumption	80
3.4.2	Plot of Residuals in Time Sequence	82
3.4.3	Plot of Residuals Versus Fitted Values	83
3.4.4	Plots of Residuals Versus Other Variables	88
3.5	Practical Interpretation of Results	89
3.5.1	A Regression Model	89
3.5.2	Comparisons Among Treatment Means	90
3.5.3	Graphical Comparisons of Means	91
3.5.4	Contrasts	92
3.5.5	Orthogonal Contrasts	94
3.5.6	Scheffé's Method for Comparing All Contrasts	96
3.5.7	Comparing Pairs of Treatment Means	97
3.5.8	Comparing Treatment Means with a Control	101
3.6	Sample Computer Output	102
3.7	Determining Sample Size	105
3.7.1	Operating Characteristic Curves	105
3.7.2	Specifying a Standard Deviation Increase	108
3.7.3	Confidence Interval Estimation Method	109
3.8	Other Examples of Single-Factor Experiments	110
3.8.1	Chocolate and Cardiovascular Health	110
3.8.2	A Real Economy Application of a Designed Experiment	110
3.8.3	Discovering Dispersion Effects	114
3.9	The Random Effects Model	116
3.9.1	A Single Random Factor	116
3.9.2	Analysis of Variance for the Random Model	117
3.9.3	Estimating the Model Parameters	118
3.10	The Regression Approach to the Analysis of Variance	125
3.10.1	Least Squares Estimation of the Model Parameters	125
3.10.2	The General Regression Significance Test	126
3.11	Nonparametric Methods in the Analysis of Variance	128
3.11.1	The Kruskal–Wallis Test	128
3.11.2	General Comments on the Rank Transformation	130
3.12	Problems	130

**4****Randomized Blocks, Latin Squares,  
and Related Designs****139**

4.1	The Randomized Complete Block Design	139
4.1.1	Statistical Analysis of the RCBD	141
4.1.2	Model Adequacy Checking	149

4.1.3	Some Other Aspects of the Randomized Complete Block Design	150
4.1.4	Estimating Model Parameters and the General Regression Significance Test	155
4.2	The Latin Square Design	158
4.3	The Graeco-Latin Square Design	165
4.4	Balanced Incomplete Block Designs	168
4.4.1	Statistical Analysis of the BIBD	168
4.4.2	Least Squares Estimation of the Parameters	172
4.4.3	Recovery of Interblock Information in the BIBD	174
4.5	Problems	177

**5****Introduction to Factorial Designs****183**

5.1	Basic Definitions and Principles	183
5.2	The Advantage of Factorials	186
5.3	The Two-Factor Factorial Design	187
5.3.1	An Example	187
5.3.2	Statistical Analysis of the Fixed Effects Model	189
5.3.3	Model Adequacy Checking	198
5.3.4	Estimating the Model Parameters	198
5.3.5	Choice of Sample Size	201
5.3.6	The Assumption of No Interaction in a Two-Factor Model	202
5.3.7	One Observation per Cell	203
5.4	The General Factorial Design	206
5.5	Fitting Response Curves and Surfaces	211
5.6	Blocking in a Factorial Design	219
5.7	Problems	225

**6****The  $2^k$  Factorial Design****233**

6.1	Introduction	233
6.2	The $2^2$ Design	234
6.3	The $2^3$ Design	241
6.4	The General $2^k$ Design	253
6.5	A Single Replicate of the $2^k$ Design	255
6.6	Additional Examples of Unreplicated $2^k$ Design	268
6.7	$2^k$ Designs are Optimal Designs	280
6.8	The Addition of Center Points to the $2^k$ Design	285
6.9	Why We Work with Coded Design Variables	290
6.10	Problems	292

**7****Blocking and Confounding in the  $2^k$  Factorial Design****304**

7.1	Introduction	304
7.2	Blocking a Replicated $2^k$ Factorial Design	305
7.3	Confounding in the $2^k$ Factorial Design	306

7.4	Confounding the $2^k$ Factorial Design in Two Blocks	306
7.5	Another Illustration of Why Blocking Is Important	312
7.6	Confounding the $2^k$ Factorial Design in Four Blocks	313
7.7	Confounding the $2^k$ Factorial Design in $2^p$ Blocks	315
7.8	Partial Confounding	316
7.9	Problems	319

## 8

### *Two-Level Fractional Factorial Designs*

320

8.1	Introduction	320
8.2	The One-Half Fraction of the $2^k$ Design	321
8.2.1	Definitions and Basic Principles	321
8.2.2	Design Resolution	323
8.2.3	Construction and Analysis of the One-Half Fraction	324
8.3	The One-Quarter Fraction of the $2^k$ Design	333
8.4	The General $2^{k-p}$ Fractional Factorial Design	340
8.4.1	Choosing a Design	340
8.4.2	Analysis of $2^{k-p}$ Fractional Factorials	343
8.4.3	Blocking Fractional Factorials	344
8.5	Alias Structures in Fractional Factorials and other Designs	349
8.6	Resolution III Designs	351
8.6.1	Constructing Resolution III Designs	351
8.6.2	Fold Over of Resolution III Fractions to Separate Aliased Effects	353
8.6.3	Plackett-Burman Designs	357
8.7	Resolution IV and V Designs	366
8.7.1	Resolution IV Designs	366
8.7.2	Sequential Experimentation with Resolution IV Designs	367
8.7.3	Resolution V Designs	373
8.8	Supersaturated Designs	374
8.9	Summary	375
8.10	Problems	376

## 9

### *Additional Design and Analysis Topics for Factorial and Fractional Factorial Designs*

394

9.1	The $3^k$ Factorial Design	395
9.1.1	Notation and Motivation for the $3^k$ Design	395
9.1.2	The $3^2$ Design	396
9.1.3	The $3^3$ Design	397
9.1.4	The General $3^k$ Design	402
9.2	Confounding in the $3^k$ Factorial Design	402
9.2.1	The $3^k$ Factorial Design in Three Blocks	403
9.2.2	The $3^k$ Factorial Design in Nine Blocks	406
9.2.3	The $3^k$ Factorial Design in $3^p$ Blocks	407
9.3	Fractional Replication of the $3^k$ Factorial Design	408
9.3.1	The One-Third Fraction of the $3^k$ Factorial Design	408
9.3.2	Other $3^{k-p}$ Fractional Factorial Designs	410

9.4	Factorials with Mixed Levels	412
9.4.1	Factors at Two and Three Levels	412
9.4.2	Factors at Two and Four Levels	414
9.5	Nonregular Fractional Factorial Designs	415
9.5.1	Nonregular Fractional Factorial Designs for 6, 7, and 8 Factors in 16 Runs	418
9.5.2	Nonregular Fractional Factorial Designs for 9 Through 14 Factors in 16 Runs	425
9.5.3	Analysis of Nonregular Fractional Factorial Designs	427
9.6	Constructing Factorial and Fractional Factorial Designs Using an Optimal Design Tool	431
9.6.1	Design Optimality Criteria	433
9.6.2	Examples of Optimal Designs	433
9.6.3	Extensions of the Optimal Design Approach	443
9.7	Problems	444

## **10** *Fitting Regression Models* **449**

10.1	Introduction	449
10.2	Linear Regression Models	450
10.3	Estimation of the Parameters in Linear Regression Models	451
10.4	Hypothesis Testing in Multiple Regression	462
10.4.1	Test for Significance of Regression	462
10.4.2	Tests on Individual Regression Coefficients and Groups of Coefficients	464
10.5	Confidence Intervals in Multiple Regression	467
10.5.1	Confidence Intervals on the Individual Regression Coefficients	467
10.5.2	Confidence Interval on the Mean Response	468
10.6	Prediction of New Response Observations	468
10.7	Regression Model Diagnostics	470
10.7.1	Scaled Residuals and PRESS	470
10.7.2	Influence Diagnostics	472
10.8	Testing for Lack of Fit	473
10.9	Problems	475

## **11** *Response Surface Methods and Designs* **478**

11.1	Introduction to Response Surface Methodology	478
11.2	The Method of Steepest Ascent	480
11.3	Analysis of a Second-Order Response Surface	486
11.3.1	Location of the Stationary Point	486
11.3.2	Characterizing the Response Surface	488
11.3.3	Ridge Systems	495
11.3.4	Multiple Responses	496
11.4	Experimental Designs for Fitting Response Surfaces	500
11.4.1	Designs for Fitting the First-Order Model	501
11.4.2	Designs for Fitting the Second-Order Model	501
11.4.3	Blocking in Response Surface Designs	507
11.4.4	Optimal Designs for Response Surfaces	511
11.5	Experiments with Computer Models	523
11.6	Mixture Experiments	530
11.7	Evolutionary Operation	540
11.8	Problems	544

**12****Robust Parameter Design and Process  
Robustness Studies****554**

12.1	Introduction	554
12.2	Crossed Array Designs	556
12.3	Analysis of the Crossed Array Design	558
12.4	Combined Array Designs and the Response Model Approach	561
12.5	Choice of Designs	567
12.6	Problems	570

**13****Experiments with Random Factors****573**

13.1	Random Effects Models	573
13.2	The Two-Factor Factorial with Random Factors	574
13.3	The Two-Factor Mixed Model	581
13.4	Sample Size Determination with Random Effects	587
13.5	Rules for Expected Mean Squares	588
13.6	Approximate <i>F</i> Tests	592
13.7	Some Additional Topics on Estimation of Variance Components	596
13.7.1	Approximate Confidence Intervals on Variance Components	597
13.7.2	The Modified Large-Sample Method	600
13.8	Problems	601

**14****Nested and Split-Plot Designs****604**

14.1	The Two-Stage Nested Design	604
14.1.1	Statistical Analysis	605
14.1.2	Diagnostic Checking	609
14.1.3	Variance Components	611
14.1.4	Staggered Nested Designs	612
14.2	The General <i>m</i> -Stage Nested Design	614
14.3	Designs with Both Nested and Factorial Factors	616
14.4	The Split-Plot Design	621
14.5	Other Variations of the Split-Plot Design	627
14.5.1	Split-Plot Designs with More Than Two Factors	627
14.5.2	The Split-Split-Plot Design	632
14.5.3	The Strip-Split-Plot Design	636
14.6	Problems	637

**15****Other Design and Analysis Topics****642**

15.1	Nonnormal Responses and Transformations	643
15.1.1	Selecting a Transformation: The Box–Cox Method	643
15.1.2	The Generalized Linear Model	645

15.2	Unbalanced Data in a Factorial Design	652
15.2.1	Proportional Data: An Easy Case	652
15.2.2	Approximate Methods	654
15.2.3	The Exact Method	655
15.3	The Analysis of Covariance	655
15.3.1	Description of the Procedure	656
15.3.2	Computer Solution	664
15.3.3	Development by the General Regression Significance Test	665
15.3.4	Factorial Experiments with Covariates	667
15.4	Repeated Measures	677
15.5	Problems	679

**Appendix****683**

<b>Table I.</b>	Cumulative Standard Normal Distribution	684
<b>Table II.</b>	Percentage Points of the <i>t</i> Distribution	686
<b>Table III.</b>	Percentage Points of the $\chi^2$ Distribution	687
<b>Table IV.</b>	Percentage Points of the <i>F</i> Distribution	688
<b>Table V.</b>	Operating Characteristic Curves for the Fixed Effects Model	
	Analysis of Variance	693
<b>Table VI.</b>	Operating Characteristic Curves for the Random Effects Model	
	Analysis of Variance	697
<b>Table VII.</b>	Percentage Points of the Studentized Range Statistic	701
<b>Table VIII.</b>	Critical Values for Dunnett's Test for Comparing Treatments	
	with a Control	703
<b>Table IX.</b>	Coefficients of Orthogonal Polynomials	705
<b>Table X.</b>	Alias Relationships for $2^{k-p}$ Fractional Factorial Designs with $k \leq 15$	
	and $n \leq 64$	706

**Bibliography****719****Index****725**