

Contents

| | |
|--|-----|
| PREFACE | iii |
| 1. DEFINITIONS AND EXAMPLES | 1 |
| 1. Bases of Zero Neighborhoods | 1 |
| 2. Examples of Topological Fields | 7 |
| 3. The Lattice $L_R(K)$ of Ring Topologies for a Field K | 16 |
| 4. Existence of Field Topologies on Infinite Fields | 20 |
| 5. Locally Bounded and Locally Unbounded Topologies | 25 |
| 6. Topological Fields and Geometry | 33 |
| Notes | 35 |
| 2. FUNDAMENTAL PROPERTIES OF ABSOLUTE-VALUED FIELDS | 37 |
| 1. Artin-Whaples' Approximation Theorem, Hensel's Lemma | 37 |
| 2. Description of Absolute Values of Some Fields | 42 |
| 3. Topological Characterizations of Absolute-Valued Fields | 50 |
| 4. Algebraic Characterizations of Absolute Values | 56 |
| 5. Extensions of Absolute-Valued Fields | 59 |
| Notes | 65 |
| 3. NORMED FIELDS AND THEIR PROPERTIES | 67 |
| 1. Examples and Properties of Normed Fields | 67 |
| 2. The Gelfand-Mazur Theorem | 70 |
| 3. Topological Characterizations of Normed Fields | 74 |
| 4. Regular and Nonregular Norms on Fields | 83 |
| 5. Norms on the Quotient Fields of Dedekind Domains | 91 |
| 6. Examples of Fields Complete with Respect to Norms Inequivalent to Absolute Values | 107 |
| Notes | 110 |
| 4. KRULL VALUATIONS | 116 |
| 1. Fundamental Properties of Valuations | 116 |
| 2. Independence Theorems | 122 |

| | |
|--|-----|
| 3. Topological Characterization of Fields with Valuations | 123 |
| 4. Extensions of Valuations | 127 |
| Notes | 128 |
| 5. TOPOLOGIES OF TYPE V | 130 |
| 1. Fundamental Properties | 130 |
| 2. Characterization of Topologies of Type V | 134 |
| 3. Minimal Locally Bounded Ring Topologies on Fields | 138 |
| 4. Strictly Minimal Topological Fields | 141 |
| Notes | 145 |
| 6. LOCALLY COMPACT TOPOLOGICAL FIELDS | 148 |
| 1. Examples of Locally Compact Fields | 148 |
| 2. Characterizations of Locally Compact Fields | 151 |
| Notes | 158 |
| 7. INDEPENDENT TOPOLOGIES ON FIELDS | 159 |
| 1. Examples of Independent Topologies | 159 |
| 2. Some Results on Independent Field Topologies | 162 |
| 3. Topologies Bounded by Suprema of V-Topologies | 167 |
| Notes | 171 |
| 8. FIELDS COMPLETE IN TWO INDEPENDENT TOPOLOGIES | 172 |
| 1. The Absolute Value Case | 172 |
| 2. The Valuation Case | 180 |
| Notes | 182 |
| 9. LOCALLY BOUNDED RING TOPOLOGIES ON FIELDS | 183 |
| 1. Several Generalizations of Valuations | 183 |
| 2. Nakano's Valuations | 186 |
| 3. Locally Bounded Ring Topologies on Some Classes of Fields | 195 |
| 4. Topological Characterizations of \mathbb{R} and \mathbb{C} | 208 |
| Note | 211 |
| 10. CONNECTED FIELDS OF ARBITRARY CHARACTERISTIC | 213 |
| Notes | 220 |
| 11. RING TOPOLOGIES ON THE RATIONAL NUMBER FIELD | 222 |
| 12. LINEAR FIELD AND RING TOPOLOGIES ON THE QUOTIENT FIELD OF ARITHMETICAL RINGS | 227 |
| 13. LOCALLY UNBOUNDED TOPOLOGIES ON FIELDS | 235 |
| 14. AN EXTENSION PROBLEM FOR TOPOLOGICAL FIELDS | 241 |
| 15. OPEN PROBLEMS | 251 |

| | |
|-----------------------------------|-----|
| APPENDIX: LINEARLY ORDERED GROUPS | 258 |
| NOTATION | 262 |
| BIBLIOGRAPHY | 263 |
| AUTHOR INDEX | 301 |
| SUBJECT INDEX | 307 |