

Contents

NOTATION	ix
PREFACE	xi
1. PRELIMINARY INFORMATION	1
1.1. Metric and Normed Spaces	1
1.2. Hilbert Spaces	6
1.3. Banach Spaces	10
1.4. Linear Operators	14
1.5. Unilateral Inverse, Generalized Inverse, and Pseudo-Inverse Operators	25
2. GENERALIZED INVERSE OPERATORS IN BANACH SPACES	29
2.1. Finite-Dimensional Operators	29
2.2. An Analog of the Schmidt Lemma for Fredholm Operators	35
2.3. Generalized Inverse Operators for Bounded Linear Fredholm Operators	39
2.4. Generalized Inverse Matrices	42
3. PSEUDOINVERSE OPERATORS IN HILBERT SPACES	47
3.1. Orthoprojectors, Their Properties and Relation to Finite-Dimensional Operators	47
3.2. An Analog of the Schmidt Lemma for Fredholm Operators	54
3.3. Left and Right Pseudoinverse Operators for Bounded Linear Fredholm Operators	60
3.4. PseudoInverse Operators for Bounded Linear Fredholm Operators	62
3.5. Inverse Operators for Fredholm Operators of Index Zero	66
3.6. A Criterion for Solvability and a Representation of Solutions of Fredholm Linear Operator Equations	68

3.7. Integral Fredholm Equations with Degenerate Kernels under Critical Conditions	73
3.8. Pseudoinverse Matrices	76
4. BOUNDARY-VALUE PROBLEMS FOR OPERATOR EQUATIONS	83
4.1. Linear Boundary-Value Problems for Fredholm Operator Equations	84
4.2. Generalized Green Operator	88
4.3. Examples	93
5. BOUNDARY-VALUE PROBLEMS FOR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS	101
5.1. Linear Boundary-Value Problems. Criterion of Solvability	101
5.2. Weakly Nonlinear Boundary-Value Problems	113
5.3. Autonomous Boundary-Value Problems	145
5.4. General Scheme of Investigation of Boundary-Value Problems . .	156
5.5. Periodic Solutions of the Mathieu, Riccati, and Van der Pol Equations	158
5.6. Differential Systems with Delay	170
6. IMPULSIVE BOUNDARY-VALUE PROBLEMS FOR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS	195
6.1. Linear Boundary-Value Problems. Criterion of Solvability	196
6.2. Generalized Green Operator for Semihomogeneous Boundary-Value Problem and Its Properties	205
6.3. Regularization of Linear Impulsive Boundary-Value Problems . .	208
6.4. Conditions for the Appearance of Solutions of Weakly Perturbed Linear Boundary-Value Problems	209
6.5. Weakly Nonlinear Boundary-Value Problems	217
6.6. Critical Case. Necessary Condition for the Existence of Solutions	221
6.7. Sufficient Condition for the Existence of Solutions. Iterative Algorithm for the Construction of Solutions	223
6.8. Critical Case of the Second Order	237

7. SOLUTIONS OF DIFFERENTIAL AND DIFFERENCE SYSTEMS BOUNDED ON THE ENTIRE REAL AXIS	257
7.1. Solutions of Linear Weakly Perturbed Systems Bounded on the Entire Real Axis	257
7.2. Nonlinear Systems	277
7.3. Solutions of Linear and Nonlinear Difference Equations Bounded on the Entire Real Axis	286
REFERENCES	305