

1	Equilibrium and Reactions	1
1.1	Forces	2
1.1.1	The parallelogram rule	2
1.1.2	Parallel forces	4
1.2	Moments	5
1.2.1	Moment from forces in a plane	6
1.2.2	Moment from forces in space	8
1.2.3	Force couples	9
1.3	Equilibrium	10
1.3.1	Virtual work of rigid bodies	11
1.3.2	Equilibrium in a plane	13
1.3.3	Distributed load	14
1.4	Support conditions	16
1.5	Reactions by equilibrium equations	19
1.5.1	Plane beams	21
1.5.2	Simple frames	24
1.5.3	Three-hinge frame	27
1.5.4	Space structures	29
1.6	Reactions by virtual work	30
1.7	Exercises	34
2	Truss Structures	39
2.1	Basic principles	41
2.1.1	Building with triangles	42
2.1.2	Counting joints and bars	43
2.1.3	Qualitative tension-compression considerations	45
2.2	Method of joints	47
2.2.1	Planar truss structures	48
2.2.2	Space trusses	53
2.3	Method of sections	54
2.3.1	Bar forces via the method of sections	55
2.3.2	Special types of planar trusses	57
2.4	Stiffness and deformation of truss structures	64
2.4.1	Axial stress and strain	64

2.4.2	Linear elastic bars	66
2.4.3	Virtual work for truss structures	67
2.4.4	Displacements of elastic truss structures	71
2.5	Finite element analysis of trusses	73
2.5.1	Elastic bar element	74
2.5.2	Finite Element Method for trusses	77
2.5.3	The <i>MiniTruss</i> program	80
2.6	Exercises	84
3	Statics of Beams and Frames	91
3.1	Internal forces and moments	93
3.2	Beams with concentrated loads	95
3.2.1	Variation of internal forces for concentrated loads	99
3.3	Beams with distributed load	104
3.3.1	Differential equations for internal forces	107
3.3.2	Maximum moment	109
3.4	Combined loads	116
3.4.1	Superposition of load cases	116
3.4.2	Superimposing the distributed load	117
3.5	Internal forces in frames	121
3.5.1	Influence of load distribution	124
3.5.2	Influence of support conditions	128
3.5.3	Three-hinge frame	131
3.5.4	Principle of the arch	133
3.6	Exercises	136
4	Deformation of Beams and Frames	143
4.1	Bending of elastic beams	144
4.1.1	Homogeneous bending	145
4.1.2	Linear kinematic relations	148
4.2	Bernoulli beam theory	151
4.2.1	Statically determinate beams	154
4.2.2	Statically indeterminate beams	159
4.3	Shear flexible beams	162
4.4	Virtual work and displacements of beams	168
4.4.1	Principle of virtual work	168
4.4.2	Displacements in elastic beams	171
4.4.3	Virtual work and displacements in frames	179
4.5	Exercises	184
5	Column Stability	189
5.1	Beam with normal force	190
5.1.1	Stiffness reduction from normal force	193
5.2	Stability of the ideal column	194

5.2.1	Equivalent column length	203
5.2.2	Buckling direction and intermediate supports	205
5.3	Design of columns	207
5.3.1	Column length and slenderness	208
5.3.2	Geometric imperfections	212
5.3.3	Stresses in column cross-sections	215
5.3.4	Perry-Robertson's column design criterion	218
5.4	Exercises	222
6	The Force Method	227
6.1	Principle of the force method	228
6.2	The general force method	233
6.2.1	Released structure	233
6.2.2	The basic steps	237
6.2.3	Summary of the force method	241
6.3	Application of the Force Method	242
6.4	The force method for frame structures	250
6.4.1	Simply supported frames	251
6.4.2	Frames with fixed supports	257
6.5	Exercises	261
7	Deformation and Element Methods for Frames	267
7.1	Stiffness of beams	268
7.1.1	Symmetric and anti-symmetric bending	269
7.1.2	Basic cases of imposed deformation	271
7.1.3	Loads on constrained beams	277
7.2	Deformation method for frames	278
7.3	Beam elements	296
7.3.1	Beam bending element	297
7.3.2	Beam-column element	299
7.3.3	Transformation to global form	305
7.4	Finite element method for frames	307
7.4.1	The <i>MiniFrame</i> program	308
7.4.2	Stability analysis of frames	312
7.5	Exercises	316
8	Stresses and Strains	321
8.1	Stress	322
8.1.1	The stress vector	322
8.1.2	General stress components	324
8.1.3	Equilibrium	329
8.2	Deformation and strain	332
8.2.1	Strain	332
8.2.2	Rotation at a point	337

8.2.3	Displacement decomposition	337
8.3	Virtual work	338
8.3.1	Equation of virtual work	338
8.3.2	Matrix and tensor notation	341
8.4	Special states of stress and strain	342
8.4.1	Plane stress and plane strain	342
8.4.2	Stress and strain transformations	343
8.4.3	Principal stresses and strains in a plane	349
8.4.4	Principal stresses in three dimensions	354
8.5	Exercises	358
9	Material Behavior	363
9.1	Elastic materials	364
9.1.1	Internal elastic energy	365
9.1.2	Linear isotropic elasticity	367
9.2	Mean and deviator components	376
9.3	Yield conditions for metals	380
9.3.1	Von Mises' yield condition	380
9.3.2	Tresca's yield condition	384
9.4	Coulomb's theory of friction materials	385
9.4.1	Critical section and stress state	386
9.4.2	Coulomb failure surface	388
9.5	Exercises	391
10	General Bending of Beams	395
10.1	Bending of non-symmetric beams	397
10.1.1	Kinematic formulation	397
10.1.2	Stresses and section forces	400
10.2	Cross-section analysis	403
10.2.1	Elastic center	403
10.2.2	Moments of inertia	411
10.2.3	Principal coordinate system	417
10.3	Axial stresses and strains	425
10.3.1	Neutral axis and line of curvature	434
10.4	Exercises	440
11	Flexure and Torsion of Beams	443
11.1	Shear stresses in beam flexure	444
11.1.1	Shear flow – Grashof's formula	445
11.1.2	Shear stress on cross-section	449
11.2	Thin-walled cross-sections in shear	455
11.2.1	Shear center	462
11.2.2	Shear flexibility	465
11.3	Torsion of circular cylinders	467

11.4 General homogeneous torsion of beams	472
11.4.1 The Prandtl stress function	475
11.5 Torsion of thin-walled beams	480
11.5.1 Open sections	480
11.5.2 Single-cell sections	484
11.5.3 Multi-cell sections	490
11.6 Exercises	493
References	499
Index	501