Contents

Introduction xi

Notation xiii

1. Unconditional and Absolute Summability in Banach Spaces 1

The Dvoretzky - Rogers Theorem 1
Absolutely convergent series are unconditionally convergent in Banach spaces, Dvoretzky - Rogers Theorem, Coincidence of absolute and unconditional summability only in finite dimensional spaces

Unconditional Convergence and the Orlicz - Pettis Theorem 4
Simple characterizations of unconditional summability, Bounded Multiplier Test, Schur's ℓ_1 Theorem, Orlicz - Pettis Theorem, Omnibus Theorem on unconditional summability

Khinchin's Inequality 9
Rademacher functions, Khinchin's Inequality, Orlicz's Theorem on unconditional summability in L_1, Span of the Rademacher functions in $L_p[0,1]$

Grothendieck's Inequality 15
Absolutely summing operators, Grothendieck's Theorem on operators from ℓ_1 to ℓ_2, Grothendieck's Inequality, Grothendieck's Theorem on finite dimensional spaces

Notes and Remarks 19

2. Fundamentals of p-Summing Operators 31

Definition 31
p-summing operators, p-summing norm

Vector-Valued Sequence Spaces 32
Strong ℓ_p sequences, Weak ℓ_p sequences, Characterization of p-summing operators

Constructions of p-Summing Operators 36
Finite rank operators, The Banach ideal of p-summing operators, Injectivity, Inclusion Theorem

Basic Examples 40
Multiplication operators, Formal inclusion operators, Diagonal operators, Embeddings of function spaces, Kernel operators

Domination and Factorization 43
Pietsch Domination Theorem, Pietsch Factorization Theorem, Operators from and to $C(K)$-spaces, 2-summing operators

Some Consequences 49
Weak compactness and complete continuity of p-summing operators, Weak Dvoretzky - Rogers Theorem, p-summing character of biadjoints and adjoints

Composition 52

Notes and Remarks 55

3. Summing Operators on $\mathcal{L}_p$-Spaces 60

$\mathcal{L}_p$-Spaces 60
Operators from $\mathcal{L}_1$-spaces to $\mathcal{L}_2$-spaces are 1-summing, Approximation in $L_p(\mu)$ and $C(K)$, $L_p(\mu)$ and $C(K)$ as basic examples of $\mathcal{L}_p$-spaces

Operators on $\mathcal{L}_\infty$-Spaces 64
Operators from $\mathcal{L}_\infty$-spaces to $\mathcal{L}_p$-spaces ($1 \le p \le 2$) are 2-summing

Some Applications 66
Quotients of $C(K)$ which are subspaces of $\mathcal{L}_1$, Coincidence of 2-summing and 1-summing operators on subspaces of $\mathcal{L}_p$ ($1 \le p \le 2$), uniqueness of unconditional basis in ℓ_1,

$L_1[0,1]$ has no unconditional basis, Coincidence of q-summing and 2-summing operators on subspaces of $\mathcal{L}_p$ for $1 \le p \le 2 < q < \infty$, Extrapolation Theorem

NOTES AND REMARKS 73

4. Operators on Hilbert Spaces and Summing Operators 76

COMPACT HILBERT SPACE OPERATORS 76
Spectral Theorem, Orthonormal Representation, Approximation Numbers

SCHATTEN - VON NEUMANN CLASSES 80
General Theory, Hilbert - Schmidt operators

HILBERT-SCHMIDT OPERATORS AND SUMMING OPERATORS 84
Coincidence with 2-summing operators, with p-summing operators, Characterization by factorization

EXTENSION PROPERTY 85
Injective Banach spaces, Π_2-Extension Theorem, Kadets - Snobar Theorem

ADJOINTS OF 2-SUMMING OPERATORS 88

NOTES AND REMARKS 90

5. p-Integral Operators 95

DEFINITION AND ELEMENTARY PROPERTIES 95
p-integral operators, Banach ideal property, strictly p-integral operators

RELATIONS TO p-SUMMING OPERATORS 97
p-integral operators are p-summing, Conditions for the converse, Further cases of coincidence

p-SUMMING OPERATORS FAILING TO BE p-INTEGRAL 99

FURTHER STRUCTURAL RESULTS 104
p-integrality of second adjoints, The case $p=1$, Composition of summing and integral operators

ORDER BOUNDEDNESS 107
Definitions, $L_p(\mu)$-valued order bounded operators are p-integral, Converse for $p=1$, Operators with p-summing adjoint

p-NUCLEAR OPERATORS 111
Various characterizations of p-nuclear operators, Banach ideal property

RELATIONS TO INTEGRAL OPERATORS 114
p-nuclear operators are p-integral, Cases of coincidence, Factorization, Composition of summing and nuclear operators

HILBERT SPACE OPERATORS 118
p-nuclear and p-integral operators on Hilbert space, Composition of 2-summing operators

NOTES AND REMARKS 119

6. Trace Duality 125

TRACE 125
Trace and trace duality in finite dimensions

DUALITY FOR SCHATTEN - VON NEUMANN CLASSES 126
Composition Theorem, Trace and duality for Schatten - von Neumann classes

BANACH IDEALS 130
Operator ideals and their norms, Banach ideals, Comparison

ADJOINT IDEALS 132
Adjoint ideals, Biadjoint criterion

MAXIMAL IDEALS 134
Ordering Banach ideals, Maximality, Maximality and adjoints, Maximality of p-integral operators, Adjoints of p-integral operators, Maximality of q-summing operators, Minimality of p-nuclear operators, Characterization of $\mathcal{L}_\infty$-spaces

APPLICATIONS 143
Lewis' Theorem, Auerbach's Lemma, John's ellipsoids, Carathéodory's Theorem
NOTES AND REMARKS 151

7. 2-Factorable Operators 154

GENERALITIES 154
The Banach ideal of p-factorable operators, adjoints, the case $p=2$
MAXIMALITY OF $[\Gamma_2, \gamma_2]$ 156
RELATIONS WITH GROTHENDIECK'S INEQUALITY 157
Matricial characterization, Preparations for Maurey's Extension Theorem
2-DOMINATED OPERATORS 162
Definition, Maximality, Trace duality of 2-dominated and 2-factorable operators, Further characterizations
NOTES AND REMARKS 166

8. Ultraproducts and Local Reflexivity 169

GENERALITIES ON ULTRAPRODUCTS 169
Ultraproduct of Banach spaces and operators, Ultrapowers, Ultraproducts of finite dimensional spaces
SOME STABILITY PROPERTIES 172
Banach lattices, $L_p(\mu)$-spaces, $C(K)$-spaces, Banach algebras and ultraproducts
ULTRAPRODUCTS AND FINITE DIMENSIONAL STRUCTURE 173
Banach spaces as subspaces of ultraproducts of finite dimensional subspaces, Representation of operators by ultraproducts of finite dimensional operators
FINITE REPRESENTABILITY 175
Concepts, Relation with ultraproducts, Application to $L_p(\mu)$- and $C(K)$-spaces
LOCAL REFLEXIVITY 177
Helly's Lemma, Principle of Local Reflexivity, Application to biduals
NOTES AND REMARKS 182

9. p-Factorable Operators 185

MAXIMALITY OF Γ_p 185
Maximality, Inclusions
DUAL IDEALS 186
Definition, Adjoints and maximality of dual ideals
q-DOMINATED OPERATORS 187
Definition and characterizations, Maximality, Trace duality of p-factorable and p^*-dominated operators
PROOF OF THE MAIN RESULT 190
Ky Fan's Lemma
APPLICATIONS 192
Characterization of complemented subspaces, subspaces and quotients of $L_p(\mu)$-spaces, Extensions of p-factorable operators
NOTES AND REMARKS 195

10. (q,p)-Summing Operators 197

SOME BASIC PROPERTIES 197
Fundamentals, Banach ideal property, $(q,2)$-summing operators and Schatten-von Neumann classes, Inclusion theorem, Dvoretzky-Rogers Theorem
OPERATORS ON $\mathcal{L}_\infty$-SPACES 199
Operators from $\mathcal{L}_\infty$ to $\mathcal{L}_p$, $p>2$, Results of interpolation and factorization for (q,p)-summing operators on $C(K)$-spaces, Coincidence of $(q,1)$-summing and (q,p)-summing operators on $C(K)$-spaces $(q>p)$, Applications
NOTES AND REMARKS 207

11. Type and Cotype: The Basics 211

Kahane's Inequality

RANDOMIZED SUMS 212
Fundamentals, Lévy's Inequality

RADEMACHER SUMS 214

RADEMACHER SUMS IN ℓ_r 216
Proof of Kahane's Inequality

TYPE AND COTYPE 217
Definitions, Type and Cotype of $\mathcal{L}_r$-spaces, Permanence Properties, Type and cotype of Lebesgue - Bochner spaces

SUMMING OPERATORS 222
Cotype and inclusion theorems for summing operators, Cotype and summing operators on $C(K)$-spaces, Orlicz's Theorem and cotype, Subspaces of L_1 having type >1

NOTES AND REMARKS 225

12. Randomized Series and Almost Summing Operators 230

RANDOMIZED SERIES 230
Almost sure summability, Standard characterizations

RADEMACHER SERIES 231
Contraction Principle, Almost sure summability and convergence in $L_p(X)$, The Banach space $\mathrm{Rad}(X)$, Relations to type and cotype

ALMOST SUMMING OPERATORS 234
Definition, p-summing operators are almost summing, Almost summing operators and cotype, Ideal properties of almost summing operators, Type 2 and almost summing operators

GAUSSIAN VARIABLES 237

APPLICATIONS TO ALMOST SUMMING OPERATORS 239
Characterization of almost summing operators using Gaussian variables, The γ-summing norm, 2-dominated and almost summing operators

SOME CONSEQUENCES 246
Kwapień's Theorem, Maurey's Extension Theorem, Applications

GAUSSIAN TYPE AND COTYPE 248
Definitions, Relations to type and cotype

THE MAUREY - ROSENTHAL THEOREM 251
Statement of the theorem, A dilation theorem, Bennett - Maurey - Nahoum decomposition of unconditionally summable sequences in L_1

NOTES AND REMARKS 255

13. K-Convexity and B-Convexity 258

K-CONVEXITY 259
Definitions, Uniform containment of ℓ_1^n's

B-CONVEXITY 261
Fundamentals, Characterization by uniform containment of ℓ_1^n's, B-convexity and duality, B-convexity and type

EQUIVALENCE OF B- AND K-CONVEXITY 267
Semigroups of operators, Beurling - Kato Theorem, Proof of the main theorem

SOME CONSEQUENCES 275
Duality between type and cotype in K-convex spaces, reflexive subspaces of L_1

NOTES AND REMARKS 280

14. Spaces with Finite Cotype **283**

Finite cotype is equivalent to non-uniform containment of ℓ_∞^n's

DVORETZKY - ROGERS AGAIN 283

FACTORING FORMAL IDENTITIES 286

THE MAIN THEOREM 289
Cotype versus finite factorization and Orlicz's Theorem, Cotype numbers, Cotype numbers of Lebesgue - Bochner spaces, Proofs

BRUNEL-SUCHESTON AFFAIRS 298
Ramsey's Theorem, Brunel - Sucheston Theorem, Invariance under spreading

NOTES AND REMARKS 303

15. Weakly Compact Operators on $C(K)$-Spaces **309**

CHARACTERIZATION OF WEAKLY COMPACT OPERATORS 309

AN APPROXIMATION SCHEME 311
Approximation by p-summing operators, Characterizations and properties

ULTRAPOWER STABILITY 313
Property (H), Type and Cotype, Rosenthal's Theorem on reflexive subspaces of L_1

SPACES VERIFYING GROTHENDIECK'S THEOREM 316
Subspaces of $C(K)$ leading to reflexive quotients, Kisliakov's Lemma, Finite cotype is a three space property, Grothendieck's Theorem for quotients of L_1 by a reflexive subspace

NOTES AND REMARKS 318

16. Type and Cotype in Banach Lattices **326**

FUNCTIONAL CALCULUS 326
Abstract M-spaces, Kakutani's Representation Theorem, Khinchin's Inequality in Banach lattices, Complexification of a Banach lattices

(q,p)-CONCAVE OPERATORS 330
Definition, Characterization via $(q,1)$-summing operators, Cotype of a Banach lattice, Maurey - Khinchin Inequality

THE RÔLE OF DISJOINTNESS 333
$(q,1)$-summing operators on $C(K)$ via disjointly supported functions, Cotype q $(2<q<\infty)$ of a Banach lattice is determined on disjoint vectors, Maurey - Khinchin Inequality and finite cotype, Order bounded and almost summing operators

TYPE AND CONVEXITY 340
(p,q)-convex operators, Duality with (p^*,q^*)-concavity, Type of Banach lattices

NOTES AND REMARKS 341

17. Local Unconditionality **344**

Unconditional basis, Unconditional basis constant

LOCAL UNCONDITIONAL STRUCTURE 344
Definitions, Banach lattices have l.u.st., X has l.u.st. iff X^{**} is complemented in a Banach lattice, l.u.st. and duality

THE GORDON - LEWIS INEQUALITY 349

GL-SPACES 350
Definitions and duality

GL-SPACES AND COTYPE 352
GL-spaces of cotype 2, Duality of type and cotype in GL-spaces

$\Lambda(2)$-SETS 355

Banach Spaces Failing (GL) 358
Failure of (GL) in $\mathcal{L}(\ell_2)$ and in Schatten - von Neumannn classes S_p, $p \neq 2$
Notes and Remarks 363

18. Summing Algebras 373

p-Summing Algebras 373
Definitions, Elementary properties and examples
Polynomial Inequalities 375
Polynomials, Norms of polynomials, Symmetric multilinear forms, Quotients of p-summing algebras
Q-Algebras and Operator Algebras 378
Q-Algebras and quotients of 1-summing algebras, Quotient algebras of uniform algebras are operator algebras
A Commutative Non-Operator Algebra 381
The Wiener algebra is not an operator algebra
Failure of the Many Variable von Neumann Inequality 383
A Commutative Non-Q Operator Algebra 386
2-Summing Algebras and Operator Algebras 387
2-summing algebras are operator algebras
Strictly p-Summing Algebras 390
Strictly p-summing algebras are uniform algebras
Notes and Remarks 393

19. Dvoretzky's Theorem and Factorization of Operators 396

Dvoretzky's Theorem, Factorizing Hilbert - Schmidt operators, Characterization of K-convexity
Fréchet Derivatives of Convex Functions 397
Group Actions and Invariant Measures 400
Actions of the Orthogonal Group 403
Action on spheres, on Grassmannians
Proof of Dvoretzky's Theorem 406
Dvoretzky - Rogers norms, Proof of the theorem, Cotype of spaces of compact operators, Type of spaces of nuclear operators
Basic Sequences 410
Special blocking of basic sequences
Factorization 412
Factorization of compact Hilbert space operators through subspaces of arbitrary Banach spaces, Application to Hilbert - Schmidt operators, Generalizations
Complementation 416
Cotype 2 numbers and γ-summing norm, Existence of finite rank projections of nice norms
K-Convexity 420
Geodesic Metric, Isoperimetric Inequality, Lévy's Lemma, X is K-convex iff it contains the ℓ_2^n's uniformly and uniformly complemented
The Isoperimetric Inequality 424
Caps, Qualitative version of the Isoperimetric Inequality, Blaschke's Selection Theorem, Spherical symmetrization
Notes and Remarks 431

References 435

Author Index 465

Subject Index 469