

# Contents

Preface

About the Author

Chapter 1 Introduction	1
1.1 Primary mechanisms of heat flow	1
1.2 Conduction	1
1.3 Application example: Silicon chip resistance calculation	4
1.4 Convection	5
1.5 Application example: Chassis panel cooled by natural convection	6
1.6 Radiation	6
1.7 Application example: Chassis panel cooled only by radiation	7
1.8 Illustrative example: Simple thermal network model for a heat sinked power transistor on a circuit board	8
1.9 Illustrative example: Thermal network circuit for a printed circuit board	9
1.10 Compact component models	10
1.11 Illustrative example: Pressure and thermal circuits for a forced air cooled enclosure	11
1.12 Illustrative example: A single chip package on a printed circuit board - the problem	14
1.13 Illustrative example: A single chip package on a printed circuit board - Fourier series analytical solution	14
1.14 Illustrative example: A single chip package on a printed circuit board - thermal network solution	15
1.15 Illustrative example: A single chip package on a printed circuit board - finite element method solution	16
1.16 Illustrative example: A single chip package on a printed circuit board - three solution methods compared	17
Exercises	18
Chapter 2 Thermodynamics of airflow	21
2.1 The first law of thermodynamics	21
2.2 Heat capacity at constant volume	23
2.3 Heat capacity at constant pressure	23
2.4 Steady gas flow as an open, steady, single stream	24
2.5 Air temperature rise: Temperature dependence	25
2.6 Air temperature rise: $T$ identified using differential forms of $\Delta T$ , $\Delta Q$	26
2.7 Air temperature rise: $T$ identified as average bulk temperature	27
Exercises	28
Chapter 3 Airflow I: Forced flow in systems	29
3.1 Preliminaries	29
3.2 Bernoulli's equation	30
3.3 Bernoulli's equation with losses	31
3.4 Fan testing	32
3.5 Estimate of fan test error accrued by measurement of downstream static pressure	33
3.6 Derivation of the "one velocity" head formula	34
3.7 Fan and system matching	35
3.8 Adding fans in series and parallel	38

3.9 Airflow resistance: Common elements	40
3.10 Airflow resistance: True circuit boards	42
3.11 Modeled circuit board elements	44
3.12 Combining airflow resistances	46
3.13 Application example: Forced air cooled enclosure	47
Exercises	53
Chapter 4 Airflow II: Forced flow in ducts, extrusions, and pin fin arrays	59
4.1 The airflow problem for channels with a rectangular cross-section	59
4.2 Entrance and exit effects for laminar and turbulent flow	60
4.3 Friction coefficient for channel flow	61
4.4 Application example: Two-sided extruded heat sink	62
4.5 A pin fin correlation	66
4.6 Application example: Pin fin problem from Khan et al.	67
4.7 Flow bypass effects according to Lee	69
4.8 Application example: Interfin air velocity calculation for a heat sink in a circuit board channel using the flow bypass method of Lee with the Muzychka and Yovanovich friction factor correlation	71
4.9 Application example: Interfin air velocity calculation for a heat sink in a circuit board channel using the flow bypass method of Lee with the <i>Handbook of Heat Transfer</i> friction factor correlation	74
4.10 Flow bypass effects according to Jonsson and Moshfegh	76
4.11 Application example: Pin fin problem from Khan et al., using the Jonsson and Moshfegh correlation, nonbypass	77
Exercises	78
Chapter 5 Airflow III: Buoyancy driven draft	79
5.1 Derivation of buoyancy driven head	79
5.2 Matching buoyancy head to system	81
5.3 Application example: Buoyancy-draft cooled enclosure	82
5.4 System models with buoyant airflow	83
Exercises	84
Chapter 6 Forced convective heat transfer I: Components	87
6.1 Forced convection from a surface	87
6.2 The Nusselt and Prandtl numbers	88
6.3 The Reynolds number	90
6.4 Classical flat plate forced convection correlation: Uniform surface temperature, laminar flow	90
6.5 Empirical correction to classical flat plate forced convection correlation, laminar flow	93
6.6 Application example: Winged aluminum heat sink	94
6.7 Classical flat plate forced convection correlation: Uniform heat rate per unit area, laminar flow	95
6.8 Classical flat plate (laminar) forced convection correlation extended to small Reynolds numbers: Uniform surface temperature	96
6.9 Circuit boards: Adiabatic heat transfer coefficients and adiabatic temperatures	98
6.10 Adiabatic heat transfer coefficient and temperature according to Faghri et al.	100
6.11 Adiabatic heat transfer coefficient and temperature for low-profile components according to Wirtz	101
6.12 Application example: Circuit board with 1.5 in. $\times$ 1.5 in. $\times$ 0.6 in. convecting modules	103

6.13 Application example: Circuit board with 0.82 in. $\times$ 0.24 in. $\times$ 0.123 in. convecting modules	110
Exercises	112
Chapter 7 Forced convective heat transfer II: Ducts, extrusions, and pin fin arrays	117
7.1 Boundary layer considerations	117
7.2 A convection/conduction model for ducts and heat sinks	118
7.3 Conversion of an isothermal heat transfer coefficient from referenced-to-inlet air to referenced-to-local air	121
7.4 Nusselt number for fully developed laminar duct flow corrected for entry length effects	122
7.5 A newer Nusselt number for laminar flow in rectangular (cross-section) ducts with entry length effects	125
7.6 Nusselt number for turbulent duct flow	125
7.7 Application example: Two-sided extruded heat sink	126
7.8 Flow bypass effects according to Jonsson and Moshfegh	129
7.9 Application example: Heat sink in a circuit board channel using the flow bypass method of Lee	132
7.10 In-line and staggered pin fin heat sinks	136
7.11 Application example: Thermal resistance of a pin fin heat sink using the correlation and problem example from Khan et al.	138
Exercises	139
Chapter 8 Natural convection heat transfer I: Plates	143
8.1 Nusselt and Grashof numbers	143
8.2 Classical flat plate correlations	144
8.3 Small device flat plate correlations	148
8.4 Application example: Vertical convecting plate	151
8.5 Application example: Vertical convecting and radiating plate	152
8.6 Vertical parallel plate correlations applicable to circuit board channels	153
8.7 Application example: Vertical card assembly	157
8.8 Recommended use of vertical channel models in sealed and vented enclosures	161
8.9 Conversion of isothermal wall channel heat transfer coefficients from referenced-to-inlet air to referenced-to-local air	162
8.10 Application example: Enclosure with circuit boards - enclosure temperatures only	163
8.11 Application example: Enclosure with circuit boards - circuit board temperatures only	168
8.12 Application example: Enclosure with circuit boards, comparison of Sections 8.10 and 8.11 approximate results with <i>CFD</i>	173
8.13 Application example: Single-circuit board enclosure with negligible circuit board radiation	173
8.14 Illustrative example: Single-circuit board enclosure with radiation exchange between interior enclosure walls and circuit board, results compared with experiment	177
8.15 Illustrative example: Metal-walled enclosure, ten <i>PCBs</i>	180
8.16 Illustrative example: Metal-walled enclosure with heat dissipation provided by several wire-wound resistors	181
Exercises	182



Chapter 9 Natural convection heat transfer II: Heat sinks	189
9.1 Heat sink geometry and some nomenclature	189
9.2 A rectangular U-channel correlation from Van de Pol and Tierney	189
9.3 Design plots representing the Van de Pol and Tierney correlation	190
9.4 A few useful formulae	197
9.5 Application example: Natural convection cooled, vertically oriented heat sink	198
9.6 Application example: Natural convection cooled, nine-fin heat sink with calculations compared to test data	199
Exercises	200
Chapter 10 Thermal radiation heat transfer	203
10.1 Blackbody radiation	203
10.2 Spatial effects and the view factor	206
10.3 Application example: View factors for finite parallel plates	211
10.4 Nonblack surfaces	213
10.5 The radiation heat transfer coefficient	215
10.6 Application example: Radiation and natural convection cooled enclosure with circuit boards - enclosure temperatures only	216
10.7 Radiation for multiple gray-body surfaces	217
10.8 Hottel script $F$ ( $\mathcal{F}$ ) method for gray-body radiation exchange	218
10.9 Application example: Gray-body circuit boards analyzed as infinite parallel plates	223
10.10 Application example: Gray-body circuit boards analyzed as finite parallel plates	225
10.11 Thermal radiation networks	226
10.12 Thermal radiation shielding for rectangular U-channels (fins)	230
10.13 Application example: Natural convection and radiation cooled, vertically oriented heat sink (see Section 9.4)	236
10.14 Application example: Natural convection and radiation cooled nine-fin heat sink - calculations compared to test data	238
10.15 Application example: Natural convection and radiation cooled nine-fin heat sink analyzed for a temperature rise not included in Figure 9.2	239
10.16 Illustrative example: Natural convection and radiation cooled nine-fin heat sink analyzed for optimum number of fins	240
Exercises	241
Chapter 11 Conduction I: Basics	247
11.1 Fourier's law of heat conduction	247
11.2 Application example: Mica insulator with thermal paste	248
11.3 Thermal conduction resistance of some simple structures	249
11.4 The one-dimensional differential equation for heat conduction	250
11.5 Application example: Aluminum core board with negligible air cooling	256
11.6 Application example: Aluminum core board with forced air cooling	256
11.7 Application example: Simple heat sink	257
11.8 Fin efficiency	258
11.9 Differential equations for more than one dimension	262
11.10 Physics of thermal conductivity of solids	263
11.11 Thermal conductivity of circuit boards (epoxy-glass laminates)	265
11.12 Application example: Epoxy-glass circuit board with copper	267
11.13 Thermal interface resistance	270

11.14 Application example: Interface resistance for an aluminum joint	272
Exercises	274
Chapter 12 Conduction II: Spreading resistance	279
12.1 The spreading problem	279
12.2 Fixed spreading angle theories	279
12.3 Circular-source, semi-infinite media solution, uniform flux, average source temperature, by Carslaw and Jaeger (1986)	282
12.4 Rectangular-source, time-dependent, semi-infinite media solution, uniform flux, peak source temperature by Joy and Schlig (1970)	284
12.5 Other circular source solutions	291
12.6 Rectangular source on rectangular, finite-media with one convecting surface: Theory	292
12.7 Rectangular source on rectangular, finite-media: Design curves	301
12.8 Application example: Heat source centered on a heat sink (Ellison, 2003)	301
12.9 Application example: IC chip on an alumina substrate	309
12.10 Rectangular source on rectangular, finite-media with two convecting surfaces: Theory	310
12.11 Exploring the difference between one-sided and two-sided Newtonian cooling	315
12.12 Method of including the effect of two different ambients to the two-sided Newtonian cooling spreading theory	316
12.13 Application example: Heat sink with two convecting sides, one finned and one flat	317
12.14 Square source on square, finite-media with one convecting surface - time dependent: Theory (Rhee and Bhatt, 2007)	319
Exercises	321
Chapter 13 Additional mathematical methods	323
13.1 Thermal networks: Steady-state theory	323
13.2 Illustrative example: A simple steady-state, thermal network problem, Gauss-Seidel and simultaneous equation solutions compared	325
13.3 Thermal networks: Time-dependent theory	328
13.4 Illustrative example: A simple time-dependent, thermal network problem	330
13.5 Finite difference theory for conduction with Newtonian cooling	334
13.6 Programming the pressure/airflow network problem	335
13.7 Finite element theory - the concept of the calculus of variations	339
13.8 Finite element theory - derivation of the one-dimensional Euler-Lagrange equation	340
13.9 Finite element theory - application of the one-dimensional Euler-Lagrange equation to a heat conduction problem	342
13.10 Finite element theory - derivation of the two-dimensional Euler-Lagrange equation	345
13.11 Finite element theory - application of the Euler-Lagrange equation to two-dimensional heat conduction	347
Appendix i: Supplemental exercises	349
Appendix ii: Physical properties of dry air at atmospheric pressure	354
Appendix iii: Radiation emissivity at room temperature	356
Appendix iv: Solution details for U-channel radiation network equations	358
Appendix v: Thermal conductivity of some common electronic packaging materials	362

Appendix <i>vi</i> : Some properties of Bessel functions	363
Appendix <i>vii</i> : Some properties of the Dirac delta function	365
Appendix <i>viii</i> : Fourier coefficients for a rectangular source	366
Appendix <i>ix</i> : Derivation of the Green's function properties for the spreading problem of a rectangular source and substrate - method A	370
Appendix <i>x</i> : Derivation of the Green's function properties for the spreading problem of a rectangular source and substrate - method B	374
Appendix <i>xi</i> : Proof of the steady-state Green's function reciprocity	377
Appendix <i>xii</i> : Nonspreading problems - unequal ambients	379
Appendix <i>xiii</i> : Finned surface to flat plate $h$ conversion	381
Appendix <i>xiv</i> : Extending the single-source problem to multiple sources and resistances	382
Appendix <i>xv</i> : Some conversion factors	385
Bibliography	387
Index	395