

---

# Contents

Preface    xiii

<b>Chapter 1. Multivariate Statistical Methods and Quality</b>	<b>1</b>
1.1 Overview of Multivariate Statistical Methods	1
1.1.1 Graphical multivariate data display and data stratification	3
1.1.2 Multivariate normal distribution and multivariate sampling distribution	3
1.1.3 Multivariate analysis of variance	4
1.1.4 Principal component analysis and factor analysis	5
1.1.5 Discriminant analysis	6
1.1.6 Cluster analysis	7
1.1.7 Mahalanobis Taguchi system (MTS)	7
1.1.8 Path analysis and structural model	8
1.1.9 Multivariate process control	10
1.2 Applications of Multivariate Statistical Methods in Business and Industry	10
1.2.1 Data mining	11
1.2.2 Chemometrics	12
1.2.3 Other applications	13
1.3 Overview of Quality Assurance and Possible Roles of Multivariate Statistical Methods	13
1.3.1 Stage 0: Impetus/ideation	13
1.3.2 Stage 1: Customer and business requirements study	15
1.3.3 Stage 2: Concept development	15
1.3.4 Stage 3: Product/service design/prototyping	15
1.3.5 Stage 4: Manufacturing process preparation/product launch	16
1.3.6 Stage 5: Production	16
1.3.7 Stage 6: Product/service consumption	17
1.3.8 Stage 7: Disposal	17
1.4 Overview of Six Sigma and Possible Roles of Multivariate Statistical Methods	18
1.4.1 Stage 1: Define the project and customer requirements (D or define step)	20
1.4.2 Stage 2: Measuring process performance	21
1.4.3 Stage 3: Analyze data and discover causes of the problem	21
1.4.4 Stage 4: Improve the process	22
1.4.5 Stage 5: Control the process	23

<b>Chapter 2. Graphical Multivariate Data Display and Data Stratification</b>	<b>25</b>
2.1 Introduction	25
2.2 Graphical Templates for Multivariate Data	26
2.2.1 Charts and graphs	26
2.2.2 Templates for displaying multivariate data	29
2.3 Data Visualization and Animation	33
2.3.1 Introduction to data visualization	33
2.4 Multivariate Data Stratification	38
2.4.1 Multi-vari chart technique	39
2.4.2 Graphical analysis of multivariate variation pattern	41
<b>Chapter 3. Introduction to Multivariate Random Variables, Normal Distribution, and Sampling Properties</b>	<b>47</b>
3.1 Overview of Multivariate Random Variables	47
3.2 Multivariate Data Sets and Descriptive Statistics	50
3.2.1 Multivariate data sets	50
3.2.2 Multivariate descriptive statistics	51
3.3 Multivariate Normal Distributions	55
3.3.1 Some properties of the multivariate normal distribution	56
3.4 Multivariate Sampling Distribution	57
3.4.1 Sampling distribution of $\bar{X}$	57
3.4.2 Sampling distribution of $S$	58
3.4.3 Central limit theorem applied to multivariate samples	58
3.4.4 Hotelling's $T^2$ distribution	59
3.4.5 Summary	60
3.5 Multivariate Statistical Inferences on Mean Vectors	60
3.5.1 Small sample multivariate hypothesis testing on a mean vector	62
3.5.2 Large sample multivariate hypothesis testing on a mean vector	63
3.5.3 Small sample multivariate hypothesis testing on the equality of two mean vectors	64
3.5.4 Large sample multivariate hypothesis testing on the equality of two mean vectors	66
3.5.5 Overview of confidence intervals and confidence regions in multivariate statistical inferences	67
3.5.6 Confidence regions and intervals for a single mean vector with small sample size	68
3.5.7 Confidence regions and intervals for a single mean vector with large sample size	70
3.5.8 Confidence regions and intervals for the difference in two population mean vectors for small samples	71
3.5.9 Confidence regions and intervals for the difference in two population mean vectors for large samples	72
3.5.10 Other Cases	73
Appendix 3A: Matrix Algebra Refresher	73
A.1 Introduction	73
A.2 Notations and basic operations	73
A.3 Matrix operations	75
<b>Chapter 4. Multivariate Analysis of Variance</b>	<b>81</b>
4.1 Introduction	81
4.2 Univariate Analysis of Variance (ANOVA)	82
4.2.1 The ANOVA table	85

4.3	Multivariate Analysis of Variance	86
4.3.1	MANOVA model	86
4.3.2	The decomposition of total variation under MANOVA model	88
4.4	MANOVA Case Study	95
<b>Chapter 5. Principal Component Analysis and Factor Analysis</b>		<b>97</b>
5.1	Introduction	97
5.2	Principal Component Analysis Based on Covariance Matrices	99
5.2.1	Two mathematical representations of principal component analysis	100
5.2.2	Properties of principal component analysis	101
5.2.3	Covariance and correlation between X and principal components Y	103
5.2.4	Principal component analysis on sample covariance matrix	103
5.3	Principal Component Analysis Based on Correlation Matrices	108
5.3.1	Principal component scores and score plots	111
5.4	Principal Component Analysis of Dimensional Measurement Data	114
5.4.1	Properties of the geometrical variation mode	117
5.4.2	Variation mode chart	118
5.4.3	Visual display and animation of principal component analysis	121
5.4.4	Applications for other multivariate data	122
5.5	Principal Component Analysis Case Studies	124
5.5.1	Improving automotive dimensional quality by using principal component analysis	124
5.5.2	Performance degradation analysis for IRLEDs (Yang and Yang, 2000)	131
5.6	Factor Analysis	141
5.6.1	Common factor analysis	143
5.6.2	Properties of common factor analysis	143
5.6.3	Parameter estimation in common factor analysis	147
5.7	Factor Rotation	148
5.7.1	Factor rotation for simple structure	149
5.7.2	Procrustes rotation	152
5.8	Factor Analysis Case Studies	152
5.8.1	Characterization of texture and mechanical properties of heat-induced soy protein gels (Kang, Matsumura, and Mori, 1991)	152
5.8.2	Procrustes factor analysis for automobile body assembly process	154
5.8.3	Hinge variation study using procrustes factor analysis	156
<b>Chapter 6. Discriminant Analysis</b>		<b>161</b>
6.1	Introduction	161
6.1.1	Discriminant analysis steps	162
6.2	Linear Discriminant Analysis for Two Normal Populations with Known Covariance Matrix	163
6.3	Linear Discriminant Analysis for Two Normal Population with Equal Covariance Matrices	167
6.4	Discriminant Analysis for Two Normal Population with Unequal Covariance Matrices	169
6.5	Discriminant Analysis for Several Normal Populations	170

6.5.1	Linear discriminant classification	170
6.5.2	Discriminant classification based on the Mahalanobis squared distances	171
6.6	Case Study: Discriminant Analysis of Vegetable Oil by Near-Infrared Reflectance Spectroscopy	175
<b>Chapter 7. Cluster Analysis</b>		<b>181</b>
7.1	Introduction	181
7.2	Distance and Similarity Measures	183
7.2.1	Euclidean distance	183
7.2.2	Standardized euclidean distance	183
7.2.3	Manhattan distance (city block distance)	184
7.2.4	Distance between clusters and linkage method	185
7.2.5	Similarity	189
7.3	Hierarchical Clustering Method	190
7.4	Nonhierarchical Clustering Method (K-Mean Method)	195
7.5	Cereal Brand Case Study	197
<b>Chapter 8. Mahalanobis Distance and Taguchi Method</b>		<b>201</b>
8.1	Introduction	201
8.2	Overview of the Mahalanobis-Taguchi System (MTS)	202
8.2.1	Stage 1: Creation of a baseline Mahalanobis space	203
8.2.2	Stage 2: Test and analysis of the Mahalanobis measure for abnormal samples	205
8.2.3	Stage 3 variable screening by using Taguchi orthogonal array experiments	206
8.2.4	Stage 4: Establish a threshold value (a cutoff MD) based on Taguchi's quality loss function and maintain a multivariate monitoring system	214
8.3	Features of the Mahalanobis-Taguchi System	216
8.4	The Mahalanobis-Taguchi System Case Study	216
8.4.1	Clutch disc inspection	217
8.5	Comments on the Mahalanobis-Taguchi System by Other Researchers and Proposed Alternative Approaches	221
8.5.1	Alternative approaches	221
<b>Chapter 9. Path Analysis and the Structural Model</b>		<b>223</b>
9.1	Introduction	223
9.2	Path Analysis and the Structural Model	225
9.2.1	How to use the path diagram and structural model	228
9.3	Advantages and Disadvantages of Path Analysis and the Structural Model	235
9.3.1	Advantages	235
9.3.2	Disadvantages	236
9.4	Path Analysis Case Studies	237
9.4.1	Path analysis model relating plastic fuel tank characteristics with its hydrocarbon permeation (Hamade, 1996)	237
9.4.2	Path analysis of a foundry process (Price and Barth, 1995)	241



<b>Chapter 10. Multivariate Statistical Process Control</b>	<b>243</b>
10.1 Introduction	243
10.2 Multivariate Control Charts for Given Targets	245
10.2.1 Decomposition of the Hotelling $T^2$	248
10.3 Two-Phase $T^2$ Multivariate Control Charts with Subgroups	251
10.3.1 Reference sample and new observations	251
10.3.2 Two-phase $T^2$ multivariate process control for subgroups	255
10.4 $T^2$ Control Chart for Individual Observations	259
10.4.1 Phase I reference sample preparation	260
10.4.2 Phase II: Process control for new observations	264
10.5 Principal Component Chart	265
<b>Appendix Probability Distribution Tables</b>	<b>271</b>
<b>References</b>	<b>291</b>
<b>Index</b>	<b>295</b>