

Contents

1	Basic Phenomena and Applications to Thin Films	1
1.1	Surface Tension at Fluid Interfaces	1
1.2	Capillary Statics	3
1.3	Liquid Transport on Microscale	7
1.4	Thermocapillary Flow in a Thin Liquid Film	11
1.4.1	Introduction to Thermocapillary Phenomena	11
1.4.2	Modeling of a Thin Film with Deforming Surface	13
1.4.3	Nondimensional Variables and Evolution Equation	16
1.5*	Stokes Flow Equations and Interfacial Boundary Conditions	21
1.6	Numerical Solution of Evolution Equations	31
1.7	Disjoining Pressure in Ultra-Thin Liquid Films	34
1.8	Notes on Literature	37
2	Coating Flows and Contact Line Models	39
2.1	Thin-Film Flow Down an Inclined Plane	39
2.1.1	Nondimensional Model Formulation	39
2.1.2	Shear-Stress Singularity	43
2.1.3	Removing the Singularity	44
2.1.4	Numerical Methods	46
2.1.5	Interface Shapes	48
2.1.6	Stokes Flow: Equations for Stream Function and Vorticity	51
2.1.7	Local Flow Near Moving Contact Line	52
2.2	Landau–Levich Problem	55
2.3	Axisymmetric Spreading of Thin Droplets	60
2.4*	Fingering Instability of Gravity-Driven Flow	61
2.4.1	Physical Phenomena	61
2.4.2	Lubrication-Type Equations in 3D	63
2.4.3	Linear Stability	64
2.4.4	Numerical Simulations	66
2.4.5	Finite-Difference Method with Nonuniform Mesh	68
2.5	Notes on Literature	69

3	Bubbles and Film Drainage	71
3.1	Static Interfaces in Channels	71
3.2	Bubble Motion in a Circular Tube	77
3.3	Bubbles in Square Tubes	81
3.4	Bubble Pressed Against a Solid Wall	82
3.5	Rupture in Liquid Films	87
3.5.1	Unstable Ultra Thin Films	87
3.5.2	Nondimensional Evolution Equation	89
3.5.3	Similarity Solution Near the Point of Rupture	92
3.6	Collapse of a Spherical Bubble	95
3.7	The Rayleigh–Plesset Equation	97
3.8	Notes on Literature	99
4	Flows in the Presence of Electric Charges and Fields	101
4.1	Electrical Double Layer	101
4.2	Electroosmotic Flow	106
4.3	Electrostatic Component of Disjoining Pressure	110
4.3.1	Equilibrium Conditions for Liquid Film on Charged Surface	110
4.3.2	Disjoining Pressure for Symmetric Electrolytes	111
4.3.3	Debye–Hückel Approximation	115
4.3.4	Application to Film Drainage	117
4.4	Electrowetting on Dielectric	120
4.5	Notes on Literature	124
5	Flows and Interface Shapes Near Structured Surfaces	125
5.1	Introduction	125
5.2	Static Droplets: Wenzel and Cassie–Baxter Configurations	127
5.3	Slip Flow in a Channel	129
5.3.1	Series Solution for Transverse Grooves	129
5.3.2	Effective Slip Length	135
5.4*	Electroosmotic Flow Over Structured Surface	138
5.5	Notes on Literature	140
6	Phase Change at Interfaces	141
6.1	Evaporation/Condensation at Flat Liquid–Vapor Interfaces	141
6.1.1	Thermodynamic Equilibrium Conditions	141
6.1.2	Equations for Evaporative Flux	145
6.2	Steady Evaporating Meniscus	149
6.3	Evaporation of Droplets into Pure Vapor	156
6.4*	More Accurate Approximations for Evaporative Flux	158
6.5	Evaporation into Air	161
6.5.1	Modeling of Vapor Diffusion	161
6.5.2	UltraThin Films in Contact with Air	164
6.5.3	Coupling Between Diffusion and Heat Transfer	165
6.6	Notes on Literature	168

7	Flows with Surfactants	169
7.1	Surfactant Spreading on Liquid Film	169
7.2*	General Equation for Interfacial Surfactant Transport	179
7.3	Models of Soluble Surfactants	183
7.4	High Concentration of Surfactants	186
7.5	Notes on Literature	187
Appendix A Selected Definitions and Theorems		
	from Thermodynamics	189
Appendix B MATLAB Codes		191
B.1	Thermocapillary flow	191
B.2	Gravity-driven flow	193
B.2.1	Unsteady evolution equation: travelling wave	193
B.2.2	Steady state equation	194
B.3	Landau-Levich problem	195
B.4	Electric double layers	196
B.5	Fourier series solution for structured surfaces	196
B.6	Local flux at the evaporating meniscus (from [103])	197
B.7	Surfactant spreading on initially uniform liquid film	199
References		203
Index		209