

Contents

Introduction	1
Part I Continuum Mechanics and Classical Materials	
1 Introduction to Continuum Mechanics	7
1.1 Introduction	7
1.2 Kinematics	7
1.2.1 Continuous Bodies. Deformations. Strain Tensors	7
1.2.2 Small Deformations. The Saint-Venant Compatibility Conditions	18
1.2.3 Transformation of Areas and Volumes. Transport Theorems	21
1.3 Principles of Continuum Mechanics	23
1.3.1 Principle of Conservation of Mass	23
1.3.2 Momentum Balance Principles	25
1.3.3 Consequences of Momentum Balance Laws	32
1.3.4 The Piola–Kirchhoff Stresses	34
1.4 Constitutive Equations	35
1.4.1 Objectivity	36
1.4.2 Principle of Material Objectivity	38
1.4.3 Fading Memory	40
2 Materials with Constitutive Equations That Are Local in Time	43
2.1 Introduction	43
2.2 Fluids. Ideal Fluids	43
2.2.1 Elastic Fluids	46
2.2.2 Newtonian Fluids. The Navier–Stokes Equations	49
2.2.3 Uniqueness of Solutions	53
2.3 Elastic Solids	55
2.3.1 Finite Elasticity	55
2.3.2 Hyperelastic Bodies	58
2.4 Linear Elasticity	62

2.4.1	Linear Elastostatics	66
2.4.2	Saint-Venant's Problem	69

Part II Continuum Thermodynamics and Constitutive Equations

3	Principles of Thermodynamics	75
3.1	Heat Equation	75
3.2	Definition of a Material as a Dynamical System	76
3.3	First Principle of Thermodynamics	78
3.4	Second Principle of Thermodynamics	80
3.4.1	The Absolute Temperature Scale	81
3.4.2	Entropy Action	83
3.5	Applications to Elastic Bodies	85
3.6	Thermodynamic Restrictions for Viscous Fluids	87
3.7	Principles of Thermodynamics for Nonsimple Materials	89
3.7.1	First Law of Thermodynamics	90
3.7.2	Second Law of Thermodynamics	96
4	Free Energies and the Dissipation Principle	99
4.1	Axiomatic Formulation of Thermodynamics	99
4.2	Minimum and Maximum Free Energies	103
5	Thermodynamics of Materials with Memory	109
5.1	Derivation of the Constitutive Equations	109
5.1.1	Required Properties of a Free Energy	115
5.1.2	Periodic Histories for General Materials	115
5.1.3	Constraints on the Nonuniqueness of the Free Energy	117
5.2	The Maximum Recoverable Work for General Materials	118
5.3	Generation of New Free Energies	120

Part III Free Energies for Materials with Linear Memory

6	A Linear Memory Model	125
6.1	A Quadratic Model for Free Energies	125
6.1.1	Constitutive Relations	128
6.1.2	Dissipation Rate	130
6.1.3	Complete Material Characterization	131
6.1.4	Linear Equilibrium Response	133
6.1.5	Time-Independent Eigenspaces	134
6.1.6	Short-Term Memory	138
6.2	Constitutive Equations in the Frequency Domain	139
6.2.1	Sinusoidal Histories for the General Theory	139
6.2.2	Properties of $\mathbb{L}'$	142
6.2.3	Frequency-Domain Representation of the History	144
6.2.4	Constitutive Equations in Terms of Frequency-Domain Quantities	145

- 6.3 The Form of the Generalized Relaxation Function 146
 - 6.3.1 Isolated Singularities 147
 - 6.3.2 Branch Cuts 148
 - 6.3.3 Essential Singularities 149
- 6.4 Minimal States in the Nonisothermal Case 150
- 6.5 Forms of the Work Function 152
- 7 Viscoelastic Solids and Fluids 155**
 - 7.1 Linear Viscoelastic Solids 155
 - 7.1.1 Thermodynamic Restrictions for Viscoelastic Solids 157
 - 7.2 Decomposition of Stress 161
 - 7.3 Equivalence and Minimal States 163
 - 7.4 State and History for Exponential-Type Relaxation Functions 165
 - 7.5 Inversion of Constitutive Relations 166
 - 7.6 Linear Viscoelastic Free Energies as Quadratic Functionals 170
 - 7.6.1 General Forms of a Free Energy in Terms of Stress 174
 - 7.6.2 The Work Function as a Free Energy 176
 - 7.7 The Relaxation Property and a Work Function Norm 178
 - 7.8 Viscoelastic Fluids 180
 - 7.9 Compressible Viscoelastic Fluids 181
 - 7.9.1 A Particular Class of Compressible Fluids 182
 - 7.9.2 Representation of Free Energies for Compressible Fluids . . . 184
 - 7.9.3 Thermodynamic Restrictions for Compressible Fluids 187
 - 7.10 Incompressible Viscoelastic Fluids 189
 - 7.10.1 Thermodynamic Restrictions for Incompressible
Viscoelastic Fluids 191
 - 7.10.2 The Mechanical Work 193
 - 7.10.3 Maximum Free Energy for Incompressible Fluids 196
- 8 Heat Conductors 199**
 - 8.1 Constitutive Equations for Rigid Heat Conductors 199
 - 8.1.1 States in Terms of $\vartheta^i(s)$ and $\mathbf{g}^i$ 201
 - 8.1.2 Constitutive Equations in Terms of States and Processes . . . 203
 - 8.1.3 Equivalent Histories and Minimal States 205
 - 8.2 Thermodynamic Constraints for Rigid Heat Conductors 207
 - 8.3 Thermal Work 208
 - 8.3.1 Integrated Histories for Isotropic Heat Conductors 209
 - 8.3.2 Finite Work Processes and w-Equivalence for States 211
 - 8.3.3 Free Energies as Quadratic Functionals for Rigid Heat
Conductors 214
 - 8.3.4 The Work Function 216

9	Free Energies on Special Classes of Material	217
9.1	The General Nonisothermal Case	217
9.1.1	The Graffi–Volterra Free Energy	217
9.1.2	Dill/Staverman–Schwarzl Free Energy	219
9.1.3	Single-Integral Quadratic Functionals of $\mathbf{I}'$	221
9.2	Free Energies for Restricted Classes of Solids	225
9.3	Free Energies for Restricted Classes of Fluids	230
9.4	Free Pseudoenergies for Restricted Classes of Rigid Heat Conductors	232
10	The Minimum Free Energy	235
10.1	Factorization of Positive Definite Tensors	236
10.1.1	The Scalar Case	239
10.2	Derivation of the Form of the Minimum Free Energy	240
10.2.1	A Variational Approach	241
10.2.2	The Wiener–Hopf Method	245
10.2.3	Histories Rather Than Relative Histories	248
10.2.4	Confirmation That ψ_m Is a Free Energy	248
10.2.5	Double Frequency Integral Form	249
10.3	Characterization of the Minimal State in the Frequency Domain	252
10.4	The Space of States and Processes	254
10.5	Limiting Properties of the Optimal Future Continuation	255
10.6	Time-Independent Eigenspaces	256
10.7	The Minimum Free Energy for Sinusoidal Histories	258
10.8	Example: Viscoelastic Materials	261
10.9	Explicit Forms of the Minimum Free Energy for Discrete Spectrum Materials	264
11	Representation of the Minimum Free Energy in the Time Domain	269
11.1	The Minimum Free Energy in Terms of Time-Domain Relative Histories	269
11.2	The Minimum Free Energy Expressed in Terms of $\mathbf{I}'$	272
12	Minimum Free Energy for Viscoelastic Solids, Fluids, and Heat Conductors	277
12.1	Maximum Recoverable Work for Solids	277
12.1.1	Minimum Free Energy for Solids	282
12.1.2	Minimum Free Energies in Terms of Stress History	286
12.2	Maximum Recoverable Work for Fluids	287
12.2.1	The Minimum Free Energy for Fluids	289
12.3	The Minimum Free Energy for Incompressible Fluids	293
12.3.1	The Minimum Free Energy in Terms of $\mathbf{I}'$	295
12.4	The Maximum Recoverable Work for Heat Conductors	298
12.4.1	The Minimum Free Energy for Heat Conductors	300
12.4.2	The Discrete Spectrum Model for Heat Conductors	303

13	The Minimum Free Energy for a Continuous-Spectrum Material	307
13.1	Introduction	307
13.2	Continuous-Spectrum Materials	308
13.3	Factorization of H for a Continuous-Spectrum Material	311
13.3.1	Properties of the Factorization Formulas	314
13.4	The Minimum Free Energy	316
13.5	An Alternative Approach	319
13.6	Minimal States	322
14	The Minimum Free Energy for a Finite-Memory Material	325
14.1	Introduction	325
14.2	Finite Memory	326
14.3	The History Dependence of the Minimum Free Energy	327
14.4	Factorization of $H(\omega)$	328
14.5	Explicit Forms of the Minimum Free Energy	331
15	A Family of Free Energies	335
15.1	Materials with Only Isolated Singularities	335
15.2	Equivalent States and the Maximum Free Energy	341
15.3	Minimization Subject to a Constraint	345
15.4	Application to a Particular Factorization	348
15.5	A Family of Free Energies That Are Functions of the Minimal State	350
15.5.1	Confirmation That ψ_f Is a Free Energy	353
15.5.2	Asymptotic Behavior, Discontinuities	354
16	Properties and Explicit Forms of Free Energies for the Case of Isolated Singularities	357
16.1	Free Energies as Discrete Quadratic Forms	357
16.1.1	The Case of Strictly Decaying Exponentials	362
16.2	Partial Orderings of the ψ_f	363
16.3	Explicit Forms for ψ_f	364
16.4	Relaxation Functions Consisting of Sums of Strictly Decaying Exponentials	365
16.5	Proposed Form of the Physical Free Energy and Dissipation	367
17	Free Energies for Nonlocal Materials	373
17.1	Second-Gradient Thermoviscoelastic Fluids	373
17.1.1	The Graffi–Volterra Free Energy	376
17.1.2	A Single-Integral Free Energy in Terms of the Minimal State	378
17.1.3	The Minimum Free Energy	379
17.2	Heat Flux in a Rigid Conductor with Nonlocal Behavior	380
17.2.1	The Graffi–Volterra Free Energy	383
17.2.2	A Free Energy in Terms of the Minimal State	384

Part IV The Dynamical Equations for Materials with Memory

18	Existence and Uniqueness	389
18.1	Introduction to Existence and Uniqueness	389
18.2	Dynamics of Viscoelastic Solids	391
18.2.1	Existence and Uniqueness of Solutions	391
18.2.2	Quasistatic Problem in Linear Viscoelasticity: Fichera's Problem	392
18.2.3	Dynamical Problem in Linear Viscoelasticity	397
19	Controllability of Thermoelastic Systems with Memory	419
19.1	The Controllability Problem: Generalities and Types	419
19.2	Exact Controllability under an Assumption on the Smallness of k	422
19.3	Exact Controllability with No Restriction on the Size of k	424
20	The Saint-Venant Problem for Viscoelastic Materials	439
20.1	Problem Description	439
20.2	A Generalized Plane Strain State	441
20.3	Analysis of the Saint-Venant Problem by Plane Cross-Section Solutions	442
20.4	Primary Solution Class	444
20.5	Secondary Solution Class	447
20.6	Solution of the Relaxed Saint-Venant Problem	448
20.7	The Saint-Venant Problem for an Isotropic and Homogeneous Cylinder	451
20.8	The Saint-Venant Principle	453
21	Exponential Decay	459
21.1	Differential Problem with Nonconvex Kernels	459
21.1.1	Transformed Problem and Some Useful Preliminaries	461
21.1.2	The Resolvent of the Kernel	463
21.1.3	Stability Results	465
22	Semigroup Theory for Abstract Equations with Memory	475
22.1	Introduction	475
22.2	The History Formulation	477
22.3	The State Formulation	478
22.4	The Semigroup in the Extended State Space	480
22.5	The Original Equation Revisited	481
22.6	Proper States	483
22.7	State versus History	486
23	Identification Problems for Integrodifferential Equations	491
23.1	Problem Specification	492
23.2	Solving the First Identification Problem	495
23.3	Solving the Second Identification Problem	501

23.4 Solving the Third Identification Problem	510
24 Dynamics of Viscoelastic Fluids	513
24.1 Introduction	513
24.2 An Initial Boundary Value Problem for an Incompressible Viscoelastic Fluid	514
24.2.1 Transformed Problem	514
24.2.2 Counterexamples to Asymptotic Stability	520
A Conventions and Some Properties of Vector Spaces	525
A.1 Notation	525
A.2 Finite-Dimensional Vector Spaces	526
A.2.1 Positive Definite Tensors	528
A.2.2 Differentiation with Respect to Vector Fields	529
A.2.3 The Vector Space <i>Sym</i>	529
B Some Properties of Functions on the Complex Plane	531
B.1 Introduction	531
B.1.1 Cauchy's Theorem and Integral Formula	532
B.1.2 Analytic Continuation	533
B.1.3 Liouville's Theorem	534
B.1.4 Singularities	534
B.1.5 Branch Points	535
B.1.6 Evaluation of Contour Integrals	536
B.2 Cauchy Integrals	538
B.2.1 Cauchy Integrals on the Real Line	541
C Fourier Transforms	545
C.1 Definitions	545
C.2 Fourier Transforms on the Complex Plane	547
C.2.1 Laplace Transforms	549
C.2.2 The Fourier Transform of Functions with Compact Support ..	549
C.2.3 Functions That Do Not Belong to $L^1 \cap L^2$	551
C.2.4 The Form of $f_{\pm}$ at Large Frequencies	552
C.2.5 Expressions for $f_{\pm}$ in Terms of f_F	553
C.3 Parseval's Formula and the Convolution Theorem	553
References	555
Index	563