

Contents

<i>Preface</i>	page ix
<i>A few words about notations</i>	xi

PART I FUNDAMENTAL CONCEPTS IN CONTINUUM MECHANICS

1 Describing the motion of a system: geometry and kinematics	3
1.1 Deformations	3
1.2 Motion and its observation (kinematics)	6
1.3 Description of the motion of a system: Eulerian and Lagrangian derivatives	10
1.4 Velocity field of a rigid body: helicoidal vector fields	13
1.5 Differentiation of a volume integral depending on a parameter	18
2 The fundamental law of dynamics	24
2.1 The concept of mass	24
2.2 Forces	30
2.3 The fundamental law of dynamics and its first consequences	32
2.4 Application to systems of material points and to rigid bodies	34
2.5 Galilean frames: the fundamental law of dynamics expressed in a non-Galilean frame	38
3 The Cauchy stress tensor and the Piola-Kirchhoff tensor. Applications	42
3.1 Hypotheses on the cohesion forces	42
3.2 The Cauchy stress tensor	45
3.3 General equations of motion	48
3.4 Symmetry of the stress tensor	50
3.5 The Piola-Kirchhoff tensor	52

4	Real and virtual powers	57
4.1	Study of a system of material points	57
4.2	General material systems: rigidifying velocities	61
4.3	Virtual power of the cohesion forces: the general case	63
4.4	Real power: the kinetic energy theorem	67
5	Deformation tensor, deformation rate tensor, constitutive laws	70
5.1	Further properties of deformations	70
5.2	The deformation rate tensor	75
5.3	Introduction to rheology: the constitutive laws	77
5.4	Appendix. Change of variable in a surface integral	87
6	Energy equations and shock equations	90
6.1	Heat and energy	90
6.2	Shocks and the Rankine–Hugoniot relations	95
PART II PHYSICS OF FLUIDS		
7	General properties of Newtonian fluids	103
7.1	General equations of fluid mechanics	103
7.2	Statics of fluids	109
7.3	Remark on the energy of a fluid	114
8	Flows of inviscid fluids	116
8.1	General theorems	116
8.2	Plane irrotational flows	120
8.3	Transsonic flows	130
8.4	Linear acoustics	134
9	Viscous fluids and thermohydraulics	137
9.1	Equations of viscous incompressible fluids	137
9.2	Simple flows of viscous incompressible fluids	138
9.3	Thermohydraulics	144
9.4	Equations in nondimensional form: similarities	146
9.5	Notions of stability and turbulence	148
9.6	Notion of boundary layer	152
10	Magnetohydrodynamics and inertial confinement of plasmas	158
10.1	The Maxwell equations and electromagnetism	159
10.2	Magnetohydrodynamics	163
10.3	The Tokamak machine	165
11	Combustion	172
11.1	Equations for mixtures of fluids	172

11.2	Equations of chemical kinetics	174
11.3	The equations of combustion	176
11.4	Stefan–Maxwell equations	178
11.5	A simplified problem: the two-species model	181
12	Equations of the atmosphere and of the ocean	185
12.1	Preliminaries	186
12.2	Primitive equations of the atmosphere	188
12.3	Primitive equations of the ocean	192
12.4	Chemistry of the atmosphere and the ocean	193
	Appendix. The differential operators in spherical coordinates	195
 PART III SOLID MECHANICS		
13	The general equations of linear elasticity	201
13.1	Back to the stress–strain law of linear elasticity: the elasticity coefficients of a material	201
13.2	Boundary value problems in linear elasticity: the linearization principle	203
13.3	Other equations	208
13.4	The limit of elasticity criteria	211
14	Classical problems of elastostatics	215
14.1	Longitudinal traction–compression of a cylindrical bar	215
14.2	Uniform compression of an arbitrary body	218
14.3	Equilibrium of a spherical container subjected to external and internal pressures	219
14.4	Deformation of a vertical cylindrical body under the action of its weight	223
14.5	Simple bending of a cylindrical beam	225
14.6	Torsion of cylindrical shafts	229
14.7	The Saint-Venant principle	233
15	Energy theorems, duality, and variational formulations	235
15.1	Elastic energy of a material	235
15.2	Duality – generalization	237
15.3	The energy theorems	240
15.4	Variational formulations	243
15.5	Virtual power theorem and variational formulations	246
16	Introduction to nonlinear constitutive laws and to homogenization	248
16.1	Nonlinear constitutive laws (nonlinear elasticity)	249

16.2 Nonlinear elasticity with a threshold (Henky's elastoplastic model)	251
16.3 Nonconvex energy functions	253
16.4 Composite materials: the problem of homogenization	255
17 Nonlinear elasticity and an application to biomechanics	259
17.1 The equations of nonlinear elasticity	259
17.2 Boundary conditions – boundary value problems	262
17.3 Hyperelastic materials	264
17.4 Hyperelastic materials in biomechanics	266
 PART IV INTRODUCTION TO WAVE PHENOMENA	
18 Linear wave equations in mechanics	271
18.1 Returning to the equations of linear acoustics and of linear elasticity	271
18.2 Solution of the one-dimensional wave equation	275
18.3 Normal modes	276
18.4 Solution of the wave equation	281
18.5 Superposition of waves, beats, and packets of waves	285
19 The soliton equation: the Korteweg–de Vries equation	289
19.1 Water-wave equations	290
19.2 Simplified form of the water-wave equations	292
19.3 The Korteweg–de Vries equation	295
19.4 The soliton solutions of the KdV equation	299
20 The nonlinear Schrödinger equation	303
20.1 Maxwell equations for polarized media	304
20.2 Equations of the electric field: the linear case	306
20.3 General case	309
20.4 The nonlinear Schrödinger equation	313
20.5 Soliton solutions of the NLS equation	316
Appendix. The partial differential equations of mechanics	319
Hints for the exercises	321
 <i>References</i>	332
<i>Index</i>	337