

Preface xiii**PART I****FUNDAMENTALS****1 Ordinary Differential Equations 3**

(E)	1.1	Ordinary Differential Equations	3
		(definitions; introductory examples)	
(E)	1.2	Initial-Value and Boundary-Value Problems	5
		(definitions; comparison of local and global analysis; examples of initial-value problems)	
(TE)	1.3	Theory of Homogeneous Linear Equations	7
		(linear dependence and independence; Wronskians; well-posed and ill-posed initial-value and boundary-value problems)	
(E)	1.4	Solutions of Homogeneous Linear Equations	11
		(how to solve constant-coefficient, equidimensional, and exact equations; reduction of order)	
(E)	1.5	Inhomogeneous Linear Equations	14
		(first-order equations; variation of parameters; Green's functions; delta function; reduction of order; method of undetermined coefficients)	
(E)	1.6	First-Order Nonlinear Differential Equations	20
		(methods for solving Bernoulli, Riccati, and exact equations; factoring; integrating factors; substitutions)	
(I)	1.7	Higher-Order Nonlinear Differential Equations	24
		(methods to reduce the order of autonomous, equidimensional, and scale-invariant equations)	

† Each section is labeled according to difficulty: (E) = easy, (I) = intermediate, (D) = difficult. A section labeled (T) indicates that the material has a theoretical rather than an applied emphasis.

(E)	1.8	Eigenvalue Problems	27
	(examples of eigenvalue problems on finite and infinite domains)		
(TE)	1.9	Differential Equations in the Complex Plane	29
	(comparison of real and complex differential equations)		
	Problems for Chapter 1		30
2 Difference Equations 36			
(E)	2.1	The Calculus of Differences	36
	(definitions; parallels between derivatives and differences, integrals, and sums)		
(E)	2.2	Elementary Difference Equations	37
	(examples of simple linear and nonlinear difference equations; gamma function; general first-order linear homogeneous and inhomogeneous equations)		
(I)	2.3	Homogeneous Linear Difference Equations	40
	(constant-coefficient equations; linear dependence and independence; Wronskians; initial-value and boundary-value problems; reduction of order; Euler equations; generating functions; eigenvalue problems)		
(I)	2.4	Inhomogeneous Linear Difference Equations	49
	(variation of parameters; reduction of order; method of undetermined coefficients)		
(E)	2.5	Nonlinear Difference Equations	53
	(elementary examples)		
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PART II

LOCAL ANALYSIS

3 Approximate Solution of Linear Differential Equations 61			
(E)	3.1	Classification of Singular Points of Homogeneous Linear Equations	62
	(ordinary, regular singular, and irregular singular points; survey of the possible kinds of behaviors of solutions)		
(E)	3.2	Local Behavior Near Ordinary Points of Homogeneous Linear Equations	66
	(Taylor series solution of first- and second-order equations; Airy equation)		
(I)	3.3	Local Series Expansions About Regular Singular Points of Homogeneous Linear Equations	68
	(methods of Fuchs and Frobenius; modified Bessel equation)		
(E)	3.4	Local Behavior at Irregular Singular Points of Homogeneous Linear Equations	76
	(failure of Taylor and Frobenius series; asymptotic relations; controlling factor and leading behavior; method of dominant balance; asymptotic series expansion of solutions at irregular singular points)		

(E)	3.5 Irregular Singular Point at Infinity	88
	(theory of asymptotic power series; optimal asymptotic approximation; behavior of modified Bessel, parabolic cylinder, and Airy functions for large positive x)	
(E)	3.6 Local Analysis of Inhomogeneous Linear Equations	103
	(illustrative examples)	
(TI)	3.7 Asymptotic Relations	107
	(asymptotic relations for oscillatory functions; Airy functions and Bessel functions; asymptotic relations in the complex plane; Stokes phenomenon; subdominance)	
(TD)	3.8 Asymptotic Series	118
	(formal theory of asymptotic power series; Stieltjes series and integrals; optimal asymptotic approximations; error estimates; outline of a rigorous theory of the asymptotic behavior of solutions to differential equations)	
	Problems for Chapter 3	136

4 Approximate Solution of Nonlinear Differential Equations 146

(E)	4.1 Spontaneous Singularities	146
	(comparison of the behaviors of solutions to linear and nonlinear equations)	
(E)	4.2 Approximate Solutions of First-Order Nonlinear Differential Equations	148
	(several examples analyzed in depth)	
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	(phase-space interpretation; classification of critical points; one- and two-dimensional phase space)	
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