Contents

Preface to the Fourth Edition ix
List of Symbols xiii

Chapter 1 **Analysis of Stress** 1

1.1 Introduction 1
1.2 Scope of Treatment 2
1.3 Definition of Stress 4
1.4 Components of Stress: Stress Tensor 5
1.5 Some Special Cases of Stress 8
1.6 Internal Force-Resultant and Stress Relations 9
1.7 Stresses on Inclined Planes in an Axially Loaded Member 11
1.8 Variation of Stress within a Body 14
1.9 Two-Dimensional Stress at a Point 17
1.10 Principal Stresses and Maximum Shear Stress in Two Dimensions 19
1.11 Mohr's Circle for Two-Dimensional Stress 20
1.12 Three-Dimensional Stress at a Point 26
1.13 Principal Stresses in Three Dimensions 29
1.14 Normal and Shear Stresses on an Oblique Plane 32
1.15 Mohr's Circle for Three-Dimensional Stress 34
1.16 Boundary Conditions in Terms of Surface Forces 37
Problems 38

Chapter 2 **Strain and Stress–Strain Relations** 51

2.1 Introduction 51
2.2 Deformation 51
2.3 Strain Defined 52
2.4 Equations of Compatibility 56
2.5 State of Strain at a Point 57
2.6 Engineering Materials 63
2.7 Stress–Strain Diagrams 64
2.8 Hooke's Law and Poisson's Ratio 66

2.9 Generalized Hooke's Law 69
2.10 Measurement of Strain: Bonded Strain Gages 72
2.11 Strain Energy 75
2.12 Strain Energy in Common Structural Members 78
2.13 Components of Strain Energy 81
2.14 Saint-Venant's Principle 83
Problems 84

Chapter 3 **Two-Dimensional Problems in Elasticity** **95**

3.1 Introduction 95
3.2 Fundamental Principles of Analysis 96
Part A—Formulation and Methods of Solution **97**
3.3 Plane Strain Problems 97
3.4 Plane Stress Problems 99
3.5 Airy's Stress Function 102
3.6 Solution of Elasticity Problems 103
3.7 Thermal Stresses 108
3.8 Basic Relations in Polar Coordinates 112
Part B—Stress Concentrations **117**
3.9 Stresses Due to Concentrated Loads 117
3.10 Stress Distribution near Concentrated Load Acting on a Beam 121
3.11 Stress Concentration Factors 123
3.12 Neuber's Diagram 127
3.13 Contact Stresses 129
Problems 136

Chapter 4 **Failure Criteria** **145**

4.1 Introduction 145
4.2 Failure 145
4.3 Failure by Yielding 146
4.4 Failure by Fracture 148
4.5 Yield and Fracture Criteria 151
4.6 Maximum Shearing Stress Theory 152
4.7 Maximum Distortion Energy Theory 153
4.8 Octahedral Shearing Stress Theory 154
4.9 Comparison of the Yielding Theories 157
4.10 Maximum Principal Stress Theory 158
4.11 Mohr's Theory 159
4.12 Coulomb–Mohr Theory 160
4.13 Introductory Fracture Mechanics 163
4.14 Failure Criteria for Metal Fatigue 167
4.15 Fatigue Life under Combined Loading 170
4.16 Impact or Dynamic Loads 172
4.17 Dynamic and Thermal Effects 175
Problems 177

Chapter 5 **Bending of Beams** **184**

5.1 Introduction 184
Part A—Exact Solutions **184**
5.2 Pure Bending of Beams of Symmetrical Cross Section 184
5.3 Pure Bending of Beams of Asymmetrical Cross Section 188
5.4 Bending of a Cantilever of Narrow Section 192
5.5 Bending of a Simply Supported, Narrow Beam 195
Part B—Approximate Solutions **197**
5.6 Elementary Theory of Bending 197
5.7 Bending and Shearing Stresses 201
5.8 Effect of Transverse Normal Stress 204
5.9 Composite Beams 206
5.10 Shear Center 212
5.11 Statically Indeterminate Systems 216
5.12 Energy Method for Deflections 219
Part C—Curved Beams **221**
5.13 Exact Solution 221
5.14 Tangential Stress. Winkler's Theory 224
5.15 Combined Tangential and Normal Stresses 228
Problems 231

Chapter 6 **Torsion of Prismatic Bars** **240**

6.1 Introduction 240
6.2 Elementary Theory of Torsion of Circular Bars 241
6.3 General Solution of the Torsion Problem 244
6.4 Prandtl's Stress Function 246
6.5 Prandtl's Membrane Analogy 252
6.6 Torsion of Thin-Walled Members of Open Cross Section 256
6.7 Torsion of Multiply Connected Thin-Walled Sections 258
6.8 Fluid Flow Analogy and Stress Concentration 262
6.9 Torsion of Restrained Thin-Walled Members of Open Cross Section 264
6.10 Curved Circular Bars: Helical Springs 267
Problems 270

Chapter 7 **Numerical Methods** **275**

7.1 Introduction 275
7.2 Finite Differences 276
7.3 Finite Difference Equations 279
7.4 Curved Boundaries 281
7.5 Boundary Conditions 284
7.6 Finite Element Method 288
7.7 Properties of a Finite Element 289
7.8 Formulation of the Finite Element Method 291
7.9 Triangular Finite Element 295

7.10 Use of Digital Computers 308
Problems 309

Chapter 8 **Axisymmetrically Loaded Members** **314**

8.1 Introduction 314
8.2 Thick-Walled Cylinders 315
8.3 Maximum Tangential Stress 320
8.4 Application of Failure Theories 321
8.5 Compound Cylinders 322
8.6 Rotating Disks of Constant Thickness 325
8.7 Rotating Disks of Variable Thickness 330
8.8 Rotating Disks of Uniform Stress 333
8.9 Thermal Stresses in Thin Disks 334
8.10 Thermal Stress in Long Circular Cylinders 336
8.11 Finite Element Solution 340
8.12 Formulation of Axisymmetric Element 341
Problems 344

Chapter 9 **Beams on Elastic Foundations** **349**

9.1 Introduction 349
9.2 General Theory 349
9.3 Infinite Beams 350
9.4 Semi-Infinite Beams 355
9.5 Finite Beams: Classification of Beams 358
9.6 Beams Supported by Equally Spaced Elastic Elements 359
9.7 Simplified Solutions for Relatively Stiff Beams 361
9.8 Solution by Finite Differences 362
9.9 Applications 364
Problems 366

Chapter 10 **Energy Methods** **369**

10.1 Introduction 369
10.2 Work Done in Deformation 369
10.3 Reciprocity Theorem 370
10.4 Castigliano's Theorem 372
10.5 Unit or Dummy Load Method 378
10.6 Crotti–Engesser Theorem 380
10.7 Statically Indeterminate Systems 382
10.8 Principle of Virtual Work 384
10.9 Principle of Minimum Potential Energy 385
10.10 Application of Trigonometric Series 387
10.11 Rayleigh–Ritz Method 391
Problems 394

Chapter 11 **Elastic Stability** **401**

11.1 Introduction 401
11.2 Critical Load 401
11.3 Buckling of a Column 403
11.4 End Conditions 405
11.5 Critical Stress in a Column 406
11.6 Allowable Stress 408
11.7 Initially Curved Members 410
11.8 Eccentrically Loaded Columns: Secant Formula 411
11.9 Energy Methods Applied to Buckling 413
11.10 Solution by Finite Differences 420
11.11 Finite Difference Solution for Unevenly Spaced Nodes 425
Problems 426

Chapter 12 **Plastic Behavior of Materials** **433**

12.1 Introduction 433
12.2 Plastic Deformation 434
12.3 True Stress–True Strain Curve in Simple Tension 434
12.4 Instability in Simple Tension 436
12.5 Plastic Deflection of Beams 438
12.6 Analysis of Perfectly Plastic Beams 440
12.7 Collapse Load of Structures 447
12.8 Elastic–Plastic Torsion 451
12.9 Elastic–Plastic Stresses in Rotating Disks 453
12.10 Plastic Stress–Strain Relations 455
12.11 Plastic Stress–Strain Increment Relations 461
12.12 Stresses in Perfectly Plastic Thick-Walled Cylinders 464
Problems 468

Chapter 13 **Plates and Shells** **472**

Part A—Bending of Thin Plates **472**
13.1 Basic Assumptions 472
13.2 Strain–Curvature Relations 473
13.3 Stress, Curvature, and Moment Relations 475
13.4 Governing Equations of Plate Deflection 476
13.5 Boundary Conditions 478
13.6 Simply Supported Rectangular Plates 481
13.7 Axisymmetrically Loaded Circular Plates 483
13.8 Deflections of Rectangular Plates by the Strain Energy Method 486
13.9 Finite Element Solution 488
Part B—Membrane Stresses in Thin Shells **491**
13.10 Basic Assumptions 491
13.11 Simple Membrane Action 491
13.12 Symmetrically Loaded Shells of Revolution 493

13.13 Some Common Cases of Shells of Revolution 495
13.14 Cylindrical Shells of General Shape 497
Problems 500

Appendix A **Indicial Notation** **503**

Appendix B **Solution of the Stress Cubic Equation** **505**

B.1 Principal Stresses 505
B.2 Direction Cosines 506

Appendix C **Moments of Composite Areas** **510**

C.1 Centroid 510
C.2 Moments of Inertia 513
C.3 Parallel-Axis Theorem 514
C.4 Principal Moments of Inertia 517

Appendix D **Tables** **523**

D.1 Average Properties of Common Engineering Materials 524
D.2 Conversion Factors: SI Units to U.S. Customary Units 526
D.3 SI Unit Prefixes 526
D.4 Deflections and Slopes of Beams 527

References **528**

Answers to Selected Problems **533**

Index **541**