
Contents

<i>Preface</i>	xv
<i>Preface to Second Edition</i>	xvii
<i>List of symbols</i>	xxi

I	Introduction	I
1.1	Objectives of the study of structural dynamics	1
1.2	Importance of vibration analysis	2
1.3	Nature of exciting forces	3
1.3.1	Dynamic forces caused by rotating machinery	3
1.3.2	Wind loads	4
1.3.3	Blast loads	4
1.3.4	Dynamic forces caused by earthquakes	5
1.3.5	Periodic and nonperiodic loads	7
1.3.6	Deterministic and nondeterministic loads	8
1.4	Mathematical modeling of dynamic systems	9
1.5	Systems of units	11
1.6	Organization of the text	12

PART I

2	Formulation of the equations of motion:	
	Single-degree-of-freedom systems	19
2.1	Introduction	19
2.2	Inertia forces	19
2.3	Resultants of inertia forces on a rigid body	21
2.4	Spring forces	26
2.5	Damping forces	29
2.6	Principle of virtual displacement	31
2.7	Formulation of the equations of motion	35
2.7.1	Systems with localized mass and localized stiffness	35
2.7.2	Systems with localized mass but distributed stiffness	37
2.7.3	Systems with distributed mass but localized stiffness	38
2.7.4	Systems with distributed stiffness and distributed mass	42
2.8	Modeling of multi-degree-of-freedom discrete parameter system	51
2.9	Effect of gravity load	53

2.10 Axial force effect	57
2.11 Effect of support motion	62
Selected readings	63
Problems	63
3 Formulation of the equations of motion:	
Multi-degree-of-freedom systems	69
3.1 Introduction	69
3.2 Principal forces in multi-degree-of-freedom dynamic system	71
3.2.1 Inertia forces	71
3.2.2 Forces arising due to elasticity	74
3.2.3 Damping forces	76
3.2.4 Axial force effects	78
3.3 Formulation of the equations of motion	79
3.3.1 Systems with localized mass and localized stiffness	80
3.3.2 Systems with localized mass but distributed stiffness	81
3.3.3 Systems with distributed mass but localized stiffness	83
3.3.4 Systems with distributed mass and distributed stiffness	89
3.4 Transformation of coordinates	102
3.5 Static condensation of stiffness matrix	106
3.6 Application of Ritz method to discrete systems	109
Selected readings	112
Problems	112
4 Principles of analytical mechanics	119
4.1 Introduction	119
4.2 Generalized coordinates	119
4.3 Constraints	124
4.4 Virtual work	127
4.5 Generalized forces	132
4.6 Conservative forces and potential energy	137
4.7 Work function	142
4.8 Lagrangian multipliers	145
4.9 Virtual work equation for dynamical systems	148
4.10 Hamilton's equation	154
4.11 Lagrange's equation	155
4.12 Constraint conditions and Lagrangian multipliers	162
4.13 Lagrange's equations for multi-degree-of-freedom systems	163
4.14 Rayleigh's dissipation function	165
Selected readings	168
Problems	168
PART 2	
5 Free vibration response: Single-degree-of-freedom system	175
5.1 Introduction	175
5.2 Undamped free vibration	175
5.2.1 Phase plane diagram	177

5.3	Free vibrations with viscous damping	186
5.3.1	Critically damped system	186
5.3.2	Overdamped system	188
5.3.3	Underdamped system	189
5.3.4	Phase plane diagram	191
5.3.5	Logarithmic decrement	192
5.4	Damped free vibration with hysteretic damping	197
5.5	Damped free vibration with coulomb damping	199
5.5.1	Phase plane representation of vibrations under Coulomb damping	202
	Selected readings	205
	Problems	205
6	Forced harmonic vibrations: Single-degree-of-freedom system	211
6.1	Introduction	211
6.2	Procedures for the solution of the forced vibration equation	212
6.3	Undamped harmonic vibration	214
6.4	Resonant response of an undamped system	218
6.5	Damped harmonic vibration	219
6.6	Complex frequency response	232
6.7	Resonant response of a damped system	237
6.8	Rotating unbalanced force	239
6.9	Transmitted motion due to support movement	244
6.10	Transmissibility and vibration isolation	249
6.11	Vibration measuring instruments	253
6.11.1	Measurement of support acceleration	253
6.11.2	Measurement of support displacement	255
6.12	Energy dissipated in viscous damping	258
6.13	Hysteretic damping	260
6.14	Complex stiffness	265
6.15	Coulomb damping	265
6.16	Measurement of damping	268
6.16.1	Free vibration decay	268
6.16.2	Forced-vibration response	269
	Selected readings	275
	Problems	275
7	Response to general dynamic loading and transient response	281
7.1	Introduction	281
7.2	Response to an Impulsive Force	281
7.3	Response to general dynamic loading	283
7.4	Response to a step function load	284
7.5	Response to a ramp function load	287
7.6	Response to a step function load with rise time	288
7.7	Response to shock loading	293
7.7.1	Rectangular pulse	293
7.7.2	Triangular pulse	297

7.7.3	Sinusoidal pulse	301
7.7.4	Effect of viscous damping	304
7.7.5	Approximate response analysis for short-duration pulses	306
7.8	Response to ground motion	307
7.8.1	Response to a short-duration ground motion pulse	313
7.9	Analysis of response by the phase plane diagram	315
	Selected readings	317
	Problems	317
8	Analysis of single-degree-of-freedom systems: Approximate and numerical methods	323
8.1	Introduction	323
8.2	Conservation of energy	325
8.3	Application of Rayleigh method to multi-degree-of-freedom systems	330
8.3.1	Flexural vibrations of a beam	335
8.4	Improved Rayleigh method	339
8.5	Selection of an appropriate vibration shape	345
8.6	Systems with distributed mass and stiffness: analysis of internal forces	349
8.7	Numerical evaluation of Duhamel's integral	352
8.7.1	Rectangular summation	353
8.7.2	Trapezoidal method	354
8.7.3	Simpson's method	355
8.8	Direct integration of the equations of motion	359
8.9	Integration based on piece-wise linear representation of the excitation	360
8.10	Derivation of general formulas	364
8.11	Constant-acceleration method	365
8.12	Newmark's β method	368
8.12.1	Average acceleration method	370
8.12.2	Linear acceleration method	372
8.13	Wilson- θ method	375
8.14	Methods based on difference expressions	377
8.14.1	Central difference method	377
8.14.2	Houbolt's method	380
8.15	Errors involved in numerical integration	381
8.16	Stability of the integration method	382
8.16.1	Newmark's β method	384
8.16.2	Wilson- θ method	387
8.16.3	Central difference method	390
8.16.4	Houbolt's method	390
8.17	Selection of a numerical integration method	390
8.18	Selection of time step	393
	Selected readings	394
	Problems	395

9 Analysis of response in the frequency domain	399
9.1 Transform methods of analysis	399
9.2 Fourier series representation of a periodic function	400
9.3 Response to a periodically applied load	402
9.4 Exponential form of Fourier series	405
9.5 Complex frequency response function	407
9.6 Fourier integral representation of a nonperiodic load	408
9.7 Response to a nonperiodic load	410
9.8 Convolution integral and convolution theorem	411
9.9 Discrete Fourier transform	413
9.10 Discrete convolution and discrete convolution theorem	416
9.11 Comparison of continuous and discrete Fourier transforms	419
9.12 Application of discrete inverse transform	426
9.13 Comparison between continuous and discrete convolution	432
9.14 Discrete convolution of an infinite- and a finite-duration waveform	437
9.15 Corrective response superposition methods	442
9.15.1 Corrective transient response based on initial conditions	444
9.15.2 Corrective periodic response based on initial conditions	448
9.15.3 Corrective responses obtained from a pair of force pulses	456
9.16 Exponential window method	459
9.17 The fast Fourier transform	464
9.18 Theoretical background to fast Fourier transform	465
9.19 Computing speed of FFT convolution	469
Selected readings	469
Problems	470

PART 3

10 Free vibration response: Multi-degree-of-freedom system	477
10.1 Introduction	477
10.2 Standard eigenvalue problem	478
10.3 Linearized eigenvalue problem and its properties	479
10.4 Expansion theorem	483
10.5 Rayleigh quotient	484
10.6 Solution of the undamped free vibration problem	488
10.7 Mode superposition analysis of free-vibration response	490
10.8 Solution of the damped free-vibration problem	496
10.9 Additional orthogonality conditions	506
10.10 Damping orthogonality	509
Selected readings	518
Problems	519
11 Numerical solution of the eigenproblem	523
11.1 Introduction	523
11.2 Properties of standard eigenvalues and eigenvectors	524

11.3	Transformation of a linearized eigenvalue problem to the standard form	526
11.4	Transformation methods	527
11.4.1	Jacobi diagonalization	529
11.4.2	Householder's transformation	534
11.4.3	QR transformation	538
11.5	Iteration methods	542
11.5.1	Vector iteration	543
11.5.2	Inverse vector iteration	546
11.5.3	Vector iteration with shifts	556
11.5.4	Subspace iteration	562
11.5.5	Lanczos iteration	564
11.6	Determinant search method	571
11.7	Numerical solution of complex eigenvalue problem	576
11.7.1	Eigenvalue problem and the orthogonality relationship	576
11.7.2	Matrix iteration for determining the complex eigenvalues	579
11.8	Semidefinite or unrestrained systems	586
11.8.1	Characteristics of an unrestrained system	586
11.8.2	Eigenvalue solution of a semidefinite system	587
11.9	Selection of a method for the determination of eigenvalues	595
	Selected readings	596
	Problems	597
12	Forced dynamic response: Multi-degree-of-freedom systems	601
12.1	Introduction	601
12.2	Normal coordinate transformation	601
12.3	Summary of mode superposition method	604
12.4	Complex frequency response	608
12.5	Vibration absorbers	615
12.6	Effect of support excitation	616
12.7	Forced vibration of unrestrained system	626
	Selected readings	631
	Problems	631
13	Analysis of multi-degree-of-freedom systems: Approximate and numerical methods	635
13.1	Introduction	635
13.2	Rayleigh–Ritz method	636
13.3	Application of Ritz method to forced vibration response	653
13.3.1	Mode superposition method	654
13.3.2	Mode acceleration method	658
13.3.3	Static condensation and Guyan's reduction	663
13.3.4	Load-dependent Ritz vectors	668
13.3.5	Application of lanczos vectors in the transformation of the equations of motion	676

13.4	Direct integration of the equations of motion	679
13.4.1	Explicit integration schemes	681
13.4.2	Implicit integration schemes	685
13.4.3	Mixed methods in direct integration	694
13.5	Analysis in the frequency domain	702
13.5.1	Frequency analysis of systems with classical mode shapes	702
13.5.2	Frequency analysis of systems without classical mode shapes	707
	Selected readings	712
	Problems	713

PART 4

14	Formulation of the equations of motion: Continuous systems	719
14.1	Introduction	719
14.2	Transverse vibrations of a beam	720
14.3	Transverse vibrations of a beam: variational formulation	722
14.4	Effect of damping resistance on transverse vibrations of a beam	729
14.5	Effect of shear deformation and rotatory inertia on the flexural vibrations of a beam	731
14.6	Axial vibrations of a bar	734
14.7	Torsional vibrations of a bar	736
14.8	Transverse vibrations of a string	738
14.9	Transverse vibrations of a shear beam	739
14.10	Transverse vibrations of a beam excited by support motion	742
14.11	Effect of axial force on transverse vibrations of a beam	746
	Selected readings	748
	Problems	749
15	Continuous systems: Free vibration response	753
15.1	Introduction	753
15.2	Eigenvalue problem for the transverse vibrations of a beam	754
15.3	General eigenvalue problem for a continuous system	757
15.3.1	Definition of the eigenvalue problem	757
15.3.2	Self-adjointness of operators in the eigenvalue problem	759
15.3.3	Orthogonality of eigenfunctions	760
15.3.4	Positive and positive definite operators	761
15.4	Expansion theorem	762
15.5	Frequencies and mode shapes for lateral vibrations of a beam	763
15.5.1	Simply supported beam	763
15.5.2	Uniform cantilever beam	766
15.5.3	Uniform beam clamped at both ends	767
15.5.4	Uniform beam with both ends free	768
15.6	Effect of shear deformation and rotatory inertia on the frequencies of flexural vibrations	772
15.7	Frequencies and mode shapes for the axial vibrations of a bar	774

15.7.1	Axial vibrations of a clamped-free bar	776
15.7.2	Axial vibrations of a free-free bar	777
15.8	Frequencies and mode shapes for the transverse vibration of a string	785
15.8.1	Vibrations of a string tied at both ends	786
15.9	Boundary conditions containing the eigenvalue	787
15.10	Free-vibration response of a continuous system	792
15.11	Undamped free transverse vibrations of a beam	794
15.12	Damped free transverse vibrations of a beam	796
	Selected readings	797
	Problems	798
16	Continuous systems: Forced-vibration response	799
16.1	Introduction	799
16.2	Normal coordinate transformation: general case of an undamped system	800
16.3	Forced lateral vibration of a beam	803
16.4	Transverse vibrations of a beam under traveling load	805
16.5	Forced axial vibrations of a uniform bar	809
16.6	Normal coordinate transformation, damped case	819
	Selected readings	825
	Problems	825
17	Wave propagation analysis	827
17.1	Introduction	827
17.2	The Phenomenon of wave propagation	828
17.3	Harmonic waves	830
17.4	One dimensional wave equation and its solution	833
17.5	Propagation of waves in systems of finite extent	839
17.6	Reflection and refraction of waves at a discontinuity in the system properties	847
17.7	Characteristics of the wave equation	851
17.8	Wave dispersion	855
	Selected readings	860
	Problems	860
PART 5		
18	Finite element method	865
18.1	Introduction	865
18.2	Formulation of the finite element equations	866
18.3	Selection of shape functions	869
18.4	Advantages of the finite element method	870
18.5	Element Shapes	870
18.5.1	One-dimensional elements	870
18.5.2	Two-dimensional elements	871
18.6	One-dimensional bar element	872

18.7	Flexural vibrations of a beam	880
18.7.1	Stiffness matrix of a beam element	883
18.7.2	Mass matrix of a beam element	884
18.7.3	Nodal applied force vector for a beam element	886
18.7.4	Geometric stiffness matrix for a beam element	886
18.7.5	Simultaneous axial and lateral vibrations	887
18.8	Stress-strain relationships for a continuum	900
18.8.1	Plane stress	902
18.8.2	Plane strain	903
18.9	Triangular element in plane stress and plane strain	904
18.10	Natural coordinates	911
18.10.1	Natural coordinate formulation for a uniaxial bar element	911
18.10.2	Natural coordinate formulation for a constant strain triangle	915
18.10.3	Natural coordinate formulation for a linear strain triangle	921
	Selected readings	926
	Problems	926
19	Component mode synthesis	931
19.1	Introduction	931
19.2	Fixed interface methods	932
19.2.1	Fixed interface normal modes	932
19.2.2	Constraint modes	933
19.2.3	Transformation of coordinates	933
19.2.4	Illustrative example	933
19.3	Free interface method	940
19.3.1	Free interface normal modes	941
19.3.2	Attachment modes	941
19.3.3	Inertia relief attachment modes	942
19.3.4	Residual flexibility attachment modes	943
19.3.5	Transformation of coordinates	944
19.3.6	Illustrative example	945
19.4	Hybrid method	951
19.4.1	Experimental determination of modal parameters	952
19.4.2	Experimental determination of the static constraint modes	957
19.4.3	Component modes and transformation of component matrices	960
19.4.4	Illustrative example	961
	Selected readings	971
	Problems	972
20	Analysis of nonlinear response	975
20.1	Introduction	975
20.2	Single-degree-of freedom system	977

20.2.1	Central difference method	979
20.2.2	Newmark's β Method	981
20.3	Errors involved in numerical integration of nonlinear systems	985
20.4	Multiple degree-of-freedom system	990
20.4.1	Explicit integration	990
20.4.2	Implicit integration	995
20.4.3	Iterations within a time step	999
Selected readings		1000
Problems		1000
<i>Answers to selected problems</i>		1003
<i>Index</i>		1019