

CONTENTS OF VOLUME 2

Introduction	xix
Notation	xxi
16 Unsymmetrical Bending	395
<i>Summary</i>	395
<i>Introduction</i>	396
16.1 <i>Product second moment of area</i>	397
16.2 <i>Principal second moments of area</i>	398
16.3 <i>Mohr's circle of second moments of area</i>	400
16.4 <i>Land's circle of second moments of area</i>	401
16.5 <i>Rotation of axes: determination of moments of area in terms of the principal values</i>	401
16.6 <i>The ellipse of second moments of area</i>	403
16.7 <i>Momental ellipse</i>	404
16.8 <i>Stress determination</i>	405
16.9 <i>Alternative procedure for stress determination</i>	405
16.10 <i>Alternative procedure using the momental ellipse</i>	407
16.11 <i>Deflections</i>	408
<i>Examples</i>	409
<i>Problems</i>	418
17 Struts	421
<i>Summary</i>	421
<i>Introduction</i>	423
17.1 <i>Euler's theory</i>	424
17.2 <i>Equivalent strut length</i>	428
17.3 <i>Comparison of Euler theory with experimental results</i>	429
17.4 <i>Euler "validity limit"</i>	430
17.5 <i>Rankine or Rankine–Gordon formula</i>	431
17.6 <i>Perry–Robertson formula</i>	432
17.7 <i>British Standard procedure (BS 449)</i>	434
17.8 <i>Struts with initial curvature</i>	434
17.9 <i>Struts with eccentric load</i>	435

17.10	<i>Laterally loaded struts</i>	439
17.11	<i>Alternative procedure for any strut-loading condition</i>	441
17.12	<i>Struts with unsymmetrical cross-sections</i>	442
	<i>Examples</i>	443
	<i>Problems</i>	450
18	Strains Beyond the Elastic Limit	454
	<i>Summary</i>	454
	<i>Introduction</i>	455
18.1	<i>Plastic bending of rectangular-sectioned beams</i>	457
18.2	<i>Shape factor – symmetrical sections</i>	458
18.3	<i>Application to I-section beams</i>	460
18.4	<i>Partially plastic bending of unsymmetrical sections</i>	460
18.5	<i>Shape factor – unsymmetrical sections</i>	462
18.6	<i>Deflections of partially plastic beams</i>	462
18.7	<i>Length of yielded area in beams</i>	462
18.8	<i>Collapse loads – plastic limit design</i>	464
18.9	<i>Residual stresses after yielding: elastic perfectly plastic material</i>	466
18.10	<i>Torsion of shafts beyond the elastic limit – plastic torsion</i>	468
18.11	<i>Angles of twist of shafts strained beyond the elastic limit</i>	470
18.12	<i>Plastic torsion of hollow tubes</i>	471
18.13	<i>Plastic torsion of case-hardened shafts</i>	472
18.14	<i>Residual stresses after yield in torsion</i>	473
18.15	<i>Plastic bending and torsion of strain-hardening materials</i>	474
18.16	<i>Residual stresses – strain-hardening materials</i>	475
18.17	<i>Influence of residual stresses on bending and torsional strengths</i>	475
18.18	<i>Plastic yielding in the eccentric loading of rectangular sections</i>	475
18.19	<i>Plastic yielding and residual stresses under axial loading with stress concentrations</i>	477
18.20	<i>Plastic yielding of axially symmetric components</i>	478
	<i>Examples</i>	480
	<i>Problems</i>	492
19	Rings, Discs and Cylinders Subjected to Rotation and Thermal Gradients	499
	<i>Summary</i>	499
19.1	<i>Thin rotating ring or cylinder</i>	500
19.2	<i>Rotating solid disc</i>	501
19.3	<i>Rotating disc with a central hole</i>	504
19.4	<i>Rotating thick cylinders or solid shafts</i>	506

19.5	<i>Rotating disc of uniform strength</i>	508
19.6	<i>Combined rotational and thermal stresses in uniform discs and thick cylinders</i>	509
	<i>Examples</i>	511
	<i>Problems</i>	519
20	Torsion of Non-circular and Thin-walled Sections	523
	<i>Summary</i>	523
20.1	<i>Rectangular sections</i>	524
20.2	<i>Narrow rectangular sections</i>	525
20.3	<i>Thin-walled open sections</i>	525
20.4	<i>Thin-walled split tube</i>	527
20.5	<i>Other solid (non-tubular) shafts</i>	527
20.6	<i>Thin-walled closed tubes of non-circular section (Bredt–Batho theory)</i>	527
20.7	<i>Thin-walled cellular sections</i>	530
20.8	<i>Membrane analogy</i>	531
20.9	<i>Effect of warping of open sections</i>	532
	<i>Examples</i>	532
	<i>Problems</i>	539
21	Experimental Stress Analysis	544
	<i>Introduction</i>	544
21.1	<i>Brittle lacquers</i>	545
21.2	<i>Strain gauges</i>	548
21.3	<i>Unbalanced bridge circuit</i>	550
21.4	<i>Null balance or balanced bridge circuit</i>	551
21.5	<i>Gauge construction</i>	551
21.6	<i>Gauge selection</i>	551
21.7	<i>Temperature compensation</i>	551
21.8	<i>Installation procedure</i>	554
21.9	<i>Basic measurement systems</i>	554
21.10	<i>D.C. and A.C. systems</i>	556
21.11	<i>Other types of strain gauge</i>	556
21.12	<i>Photoelasticity</i>	557
21.13	<i>Plane-polarised light – basic polariscope arrangements</i>	558
21.14	<i>Temporary birefringence</i>	560
21.15	<i>Production of fringe patterns</i>	560
21.16	<i>Interpretation of fringe patterns</i>	562

21.17	<i>Calibration</i>	563
21.18	<i>Fractional fringe order determination – compensation techniques</i>	563
21.19	<i>Isoclinics – circular polarisation</i>	565
21.20	<i>Stress separation procedures</i>	566
21.21	<i>Three-dimensional photoelasticity</i>	567
21.22	<i>Reflective coating technique</i>	567
21.23	<i>Other methods of strain measurement</i>	567
	<i>Bibliography</i>	569
22	Circular Plates and Diaphragms	570
	<i>Summary</i>	570
	<i>A. CIRCULAR PLATES</i>	572
22.1	<i>Stresses</i>	572
22.2	<i>Bending moments</i>	574
22.3	<i>General equation for slope and deflection</i>	574
22.4	<i>General case of a circular plate or diaphragm subjected to combined uniformly distributed load q (pressure) and central concentrated load F</i>	576
22.5	<i>Uniformly loaded circular plate with edges clamped</i>	577
22.6	<i>Uniformly loaded circular plate with edges freely supported</i>	579
22.7	<i>Circular plate with central concentrated load F and edges clamped</i>	580
22.8	<i>Circular plate with central concentrated load F and edges freely supported</i>	581
22.9	<i>Circular plate subjected to a load F distributed round a circle</i>	583
22.10	<i>Application to the loading of annular rings</i>	584
22.11	<i>Stress distributions in circular plates and diaphragms subjected to lateral pressures</i>	585
22.12	<i>Discussion of results – limitations of theory</i>	586
22.13	<i>Other loading cases of practical importance</i>	588
	<i>B. BENDING OF RECTANGULAR PLATES</i>	589
22.14	<i>Rectangular plates with simply supported edges carrying uniformly distributed loads</i>	589
22.15	<i>Rectangular plates with clamped edges carrying uniformly distributed loads</i>	590
	<i>Examples</i>	591
	<i>Problems</i>	593
23	Introduction to Advanced Elasticity Theory	595
23.1	<i>Types of stress</i>	595
23.2	<i>The cartesian stress components: notation and sign convention</i>	595
23.2.1	<i>Sign conventions</i>	596

23.3	<i>The state of stress at a point</i>	596
23.4	<i>Principal stresses and strains in three dimensions – Mohr's circle representation</i>	599
23.5	<i>The combined Mohr diagram for three-dimensional stress and strain systems</i>	600
23.6	<i>Application of the combined circle to two-dimensional stress systems</i>	603
23.7	<i>Graphical construction for the state of stress at a point</i>	604
23.8	<i>Construction for the state of strain on a general strain plane</i>	605
23.9	<i>The stress equations of equilibrium</i>	606
23.10	<i>Principal stresses in a three-dimensional cartesian stress system</i>	611
	23.10.1 <i>Solution of cubic equations</i>	611
23.11	<i>Octahedral planes and stresses</i>	612
23.12	<i>Deviatoric stresses</i>	614
23.13	<i>Deviatoric strains</i>	616
23.14	<i>Plane stress and plane strain</i>	616
	23.14.1 <i>Plane stress</i>	617
	23.14.2 <i>Plane strain</i>	618
23.15	<i>The stress–strain relations</i>	618
23.16	<i>The strain–displacement relationships</i>	619
23.17	<i>The strain equations of transformation</i>	621
23.18	<i>Compatibility</i>	622
	<i>Examples</i>	623
	<i>Problems</i>	630
Appendix.	Typical mechanical and physical properties for engineering materials	637
	Index	639

CONTENTS OF VOLUME 1

Introduction	xix
Notation	xxi
1 Simple Stress and Strain	1
1.1 Load	1
1.2 Direct stress (σ)	2
1.3 Direct strain (ϵ)	2
1.4 Sign convention for direct stress and strain	3
1.5 Elastic materials – Hooke's law	3
1.6 Modulus of elasticity – Young's modulus	3
1.7 Tensile test	4
1.8 Ductile materials	6
1.9 Brittle materials	6
1.10 Poisson's ratio	6
1.11 Shear stress	8
1.12 Shear strain	8
1.13 Modulus of rigidity	8
1.14 Double shear	9
1.15 Allowable working stress – factor of safety	9
1.16 Load factor	10
1.17 Temperature stresses	10
1.18 Stress concentrations – stress concentration factor	11
1.19 Toughness	11
1.20 Creep and fatigue	11
Examples	14
Problems	21
Bibliography	22
2 Compound Bars	23
Summary	23
2.1 Compound bars subjected to external load	24
2.2 Compound bars – “equivalent” or “combined” modulus	25

2.3	<i>Compound bars subjected to temperature change</i>	26
2.4	<i>Compound bar (tube and rod)</i>	28
2.5	<i>Compound bars subjected to external load and temperature effects</i>	30
2.6	<i>Compound thick cylinders subjected to temperature changes</i>	30
	<i>Examples</i>	30
	<i>Problems</i>	36
3	Shearing Force and Bending Moment Diagrams	37
	<i>Summary</i>	37
3.1	<i>Shearing force and bending moment</i>	37
3.1.1	<i>Shearing force (S.F.) sign convention</i>	38
3.1.2	<i>Bending moment (B.M.) sign convention</i>	38
3.2	<i>S.F. and B.M. diagrams for beams carrying concentrated loads only</i>	39
3.3	<i>S.F. and B.M. diagrams for uniformly distributed loads</i>	42
3.4	<i>S.F. and B.M. diagrams for combined concentrated and uniformly distributed loads</i>	43
3.5	<i>Points of contraflexure</i>	44
3.6	<i>Relationship between S.F. Q, B.M. M, and intensity of loading w</i>	44
3.7	<i>S.F. and B.M. diagrams for an applied couple or moment</i>	46
3.8	<i>S.F. and B.M. diagrams for inclined loads</i>	49
3.9	<i>Graphical construction of S.F. and B.M. diagrams</i>	49
3.10	<i>S.F. and B.M. diagrams for beams carrying distributed loads of increasing value</i>	51
3.11	<i>S.F. at points of application of concentrated loads</i>	51
	<i>Examples</i>	51
	<i>Problems</i>	55
4	Bending	57
	<i>Summary</i>	57
	<i>Introduction</i>	58
4.1	<i>Simple bending theory</i>	59
4.2	<i>Neutral axis</i>	61
4.3	<i>Section modulus</i>	62
4.4	<i>Second moment of area</i>	63
4.5	<i>Bending of composite or flitched beams</i>	65
4.6	<i>Reinforced concrete beams – simple tension reinforcement</i>	66
4.7	<i>Skew loading</i>	68
4.8	<i>Combined bending and direct stress – eccentric loading</i>	68
4.9	<i>“Middle-quarter” and “middle-third” rules</i>	70
4.10	<i>Shear stresses owing to bending</i>	72

4.11	<i>Strain energy in bending</i>	72
	<i>Examples</i>	72
	<i>Problems</i>	81
5	Slope and Deflection of Beams	84
	<i>Summary</i>	84
	<i>Introduction</i>	86
5.1	<i>Relationship between loading, S.F., B.M., slope and deflection</i>	86
5.2	<i>Direct integration method</i>	89
5.3	<i>Macaulay's method</i>	94
5.4	<i>Macaulay's method for u.d.l.s</i>	97
5.5	<i>Macaulay's method for beams with u.d.l. applied over part of the beam</i>	98
5.6	<i>Macaulay's method for couple applied at a point</i>	98
5.7	<i>Mohr's area-moment method</i>	100
5.8	<i>Principle of superposition</i>	104
5.9	<i>Energy method</i>	104
5.10	<i>Maxwell's theorem of reciprocal displacements</i>	104
5.11	<i>Continuous beams – Clapeyron's three-moment equation</i>	107
	<i>Examples</i>	110
	<i>Problems</i>	122
6	Built-in Beams	124
	<i>Summary</i>	124
	<i>Introduction</i>	124
6.1	<i>Built-in beam carrying central concentrated load</i>	125
6.2	<i>Built-in beam carrying uniformly distributed load across the span</i>	126
6.3	<i>Built-in beam carrying concentrated load offset from the centre</i>	127
6.4	<i>Built-in beam carrying a non-uniform distributed load</i>	128
6.5	<i>Advantages and disadvantages of built-in beams</i>	129
6.6	<i>Effect of movement of supports</i>	130
	<i>Examples</i>	131
	<i>Problems</i>	136
7	Shear Stress Distribution	137
	<i>Summary</i>	137
	<i>Introduction</i>	138
7.1	<i>Distribution of shear stress due to bending</i>	139
7.2	<i>Application to rectangular sections</i>	140

7.3	<i>Application to I-section beams</i>	141
7.3.1	<i>Vertical shear in the web</i>	142
7.3.2	<i>Vertical shear in the flanges</i>	142
7.3.3	<i>Horizontal shear in the flanges</i>	143
7.4	<i>Application to circular sections</i>	144
7.5	<i>Shear centre</i>	146
	<i>Examples</i>	147
	<i>Problems</i>	153
8	Torsion	156
	<i>Summary</i>	156
8.1	<i>Simple torsion theory</i>	157
8.2	<i>Polar second moment of area</i>	159
8.3	<i>Shear stress and shear strain in shafts</i>	160
8.4	<i>Section modulus</i>	161
8.5	<i>Torsional rigidity</i>	162
8.6	<i>Torsion of hollow shafts</i>	162
8.7	<i>Torsion of thin-walled tubes</i>	163
8.8	<i>Composite shafts – series connection</i>	163
8.9	<i>Composite shafts – parallel connection</i>	164
8.10	<i>Principal stresses</i>	164
8.11	<i>Strain energy in torsion</i>	166
8.12	<i>Variation of data along shaft length – torsion of tapered shafts</i>	166
8.13	<i>Combined bending and torsion – equivalent bending moment</i>	167
8.14	<i>Combined bending and torsion – equivalent torque</i>	168
8.15	<i>Combined bending, torsion and direct thrust</i>	169
	<i>Examples</i>	169
	<i>Problems</i>	174
9	Thin Cylinders and Shells	177
	<i>Summary</i>	177
9.1	<i>Thin cylinders under internal pressure</i>	177
9.1.1	<i>Hoop or circumferential stress</i>	178
9.1.2	<i>Longitudinal stress</i>	178
9.1.3	<i>Changes in dimensions</i>	179
9.2	<i>Thin spherical shell under internal pressure</i>	181
9.2.1	<i>Change in internal volume</i>	181
9.3	<i>Vessels subjected to fluid pressure</i>	182
9.4	<i>Cylindrical vessel with hemispherical ends</i>	182

9.5	<i>Effects of end plates and joints</i>	184
9.6	<i>Wire-wound thin cylinders</i>	184
	<i>Examples</i>	186
	<i>Problems</i>	192
10	Thick cylinders	194
	<i>Summary</i>	194
10.1	<i>Difference in treatment between thin and thick cylinders – basic assumptions</i>	195
10.2	<i>Development of the Lamé theory</i>	196
10.3	<i>Thick cylinder – internal pressure only</i>	198
10.4	<i>Longitudinal stress</i>	199
10.5	<i>Maximum shear stress</i>	200
10.6	<i>Change of cylinder dimensions</i>	200
10.7	<i>Comparison with thin cylinder theory</i>	201
10.8	<i>Graphical treatment – Lamé line</i>	202
10.9	<i>Compound cylinders</i>	203
10.10	<i>Compound cylinders – graphical treatment</i>	205
10.11	<i>Shrinkage allowance</i>	206
10.12	<i>Hub on solid shaft</i>	208
10.13	<i>Force fits</i>	208
10.14	<i>Compound cylinder – different materials</i>	209
10.15	<i>Uniform heating of compound cylinders of different materials</i>	210
10.16	<i>Plastic yielding – “auto-frettage”</i>	212
10.17	<i>Failure theories – yield criteria</i>	212
10.18	<i>Plastic theory – collapse pressure</i>	213
10.19	<i>Wire-wound thick cylinders</i>	215
	<i>Examples</i>	217
	<i>Problems</i>	233
11	Strain Energy	236
	<i>Summary</i>	236
	<i>Introduction</i>	238
11.1	<i>Strain energy – tension or compression</i>	239
11.2	<i>Strain energy – shear</i>	241
11.3	<i>Strain energy – bending</i>	242
11.4	<i>Strain energy – torsion</i>	243
11.5	<i>Strain energy of a three-dimensional principal stress system</i>	243
11.6	<i>Volumetric or dilatational strain energy</i>	244
11.7	<i>Shear or distortional strain energy</i>	244

11.8	<i>Suddenly applied loads</i>	244
11.9	<i>Impact loads – axial load application</i>	245
11.10	<i>Impact loads – bending applications</i>	247
11.11	<i>Castigliano's first theorem for deflection</i>	248
11.12	<i>Application of Castigliano's theorem to angular movements</i>	250
11.13	<i>"Unit-load" method</i>	250
11.14	<i>Shear deflection</i>	251
	<i>Examples</i>	255
	<i>Problems</i>	272
12	Springs	276
	<i>Summary</i>	276
	<i>Introduction</i>	278
12.1	<i>Close-coiled helical spring subjected to axial load W</i>	278
12.2	<i>Close-coiled helical spring subjected to axial torque T</i>	279
12.3	<i>Open-coiled helical spring subjected to axial load W</i>	280
12.4	<i>Open-coiled helical spring subjected to axial torque T</i>	283
12.5	<i>Springs in series</i>	284
12.6	<i>Springs in parallel</i>	285
12.7	<i>Leaf or carriage spring: semi-elliptic</i>	285
12.8	<i>Leaf or carriage spring: quarter-elliptic</i>	289
12.9	<i>Spiral spring</i>	290
	<i>Examples</i>	293
	<i>Problems</i>	299
13	Complex Stresses	302
	<i>Summary</i>	302
13.1	<i>Stresses on oblique planes</i>	303
13.2	<i>Material subjected to pure shear</i>	303
13.3	<i>Material subjected to two mutually perpendicular direct stresses</i>	305
13.4	<i>Material subjected to combined direct and shear stresses</i>	305
13.5	<i>Principal plane inclination in terms of the associated principal stress</i>	307
13.6	<i>Graphical solution – Mohr's stress circle</i>	308
13.7	<i>Three-dimensional stresses – graphical representation</i>	310
	<i>Examples</i>	312
	<i>Problems</i>	329
14	Complex Strain and the Elastic Constants	331
	<i>Summary</i>	331
14.1	<i>Linear strain for tri-axial stress state</i>	331

14.2	<i>Principal strains in terms of stresses</i>	332
14.3	<i>Principal stresses in terms of strains – two-dimensional stress system</i>	332
14.4	<i>Bulk modulus K</i>	333
14.5	<i>Volumetric strain</i>	333
14.6	<i>Volumetric strain for unequal stresses</i>	334
14.7	<i>Change in volume of circular bar</i>	335
14.8	<i>Effect of lateral restraint</i>	335
14.9	<i>Relationship between the elastic constants E, G, K and ν</i>	337
14.10	<i>Strains on an oblique plane</i>	340
14.11	<i>Principal strain – Mohr's strain circle</i>	342
14.12	<i>Mohr's strain circle – alternative derivation from the general stress equations</i>	344
14.13	<i>Relationship between Mohr's stress and strain circles</i>	346
14.14	<i>Construction of strain circle from three known strains (McClintock method) – rosette analysis</i>	348
14.15	<i>Analytical determination of principal strains from rosette readings</i>	350
14.16	<i>Strain energy of three-dimensional stress system</i>	352
	<i>Examples</i>	354
	<i>Problems</i>	364
15	Theories of Elastic Failure	369
	<i>Summary</i>	369
	<i>Introduction</i>	369
15.1	<i>Maximum principal stress theory</i>	370
15.2	<i>Maximum shear stress theory</i>	370
15.3	<i>Maximum principal strain theory</i>	371
15.4	<i>Maximum total strain energy per unit volume theory</i>	371
15.5	<i>Maximum shear strain energy per unit volume (or distortion energy) theory</i>	371
15.6	<i>Mohr's modified shear stress theory for brittle materials</i>	372
15.7	<i>Graphical representation of failure theories for two-dimensional stress systems (one principal stress zero)</i>	374
15.8	<i>Graphical solution of two-dimensional theory of failure problems</i>	378
15.9	<i>Graphical representation of the failure theories for three-dimensional stress systems</i>	
	15.9.1 <i>Ductile materials</i>	379
	15.9.2 <i>Brittle materials</i>	381
15.10	<i>Conclusions</i>	381
	<i>Examples</i>	381
	<i>Problems</i>	391
	Appendix. Typical mechanical and physical properties for engineering materials	xxiii
	Index	xxv