

CONTENTS

PREFACE	<i>page</i>	v
NOTE ON S.I. UNITS		vii

CHAPTER 1 INTRODUCTION

1.1	Phenomenological Nature of Theory of Plasticity	1
1.2	The Load-Extension Diagram in Simple Tension	1
1.3	The True Stress-Strain Diagram in Simple Tension	4
1.4	An Example of the Use of the Volume Constancy Equation: Drifting, Low-speed Plate Perforation or Hole Flanging	6
1.5	Some Deviations from the Stress-Strain Curves described Above	8
1.6	Frictionless Compression: Homogeneous Compression	10
1.7	General Approach to Stress Analysis in Elasticity and Plasticity	12
1.7.1	The Theory of Elasticity in Isotropic Solids	12
1.7.2	The Theory of Plasticity	13
1.8	Empirical Equations to Stress-Strain Curves	15
1.9	Empirical Equations and the Maximum Load in Simple Tension	17
1.10	Compression of a Work-hardening Material: Adiabatic Temperature Rise	18
1.11	Brittle and Ductile Materials: Cleavage- and Shear-type Fractures: Transition Temperature	19
1.12	The Effects of Hydrostatic Pressure	20
1.13	Strain Rate in Simple Uniaxial Compression and Tension	22
1.14	Cold- and Hot-working: Recrystallization and Homologous Temperature	23
1.15	Strain Rate in Relation to Recrystallization Temperature	27
1.16	A Point of Reversal in Compression	27
1.17	The Ratio of Dynamic to Static Yield Stress	28
1.18	Some Test Techniques for Providing Uniaxial Compression and Simple Stress-Strain Data with Special Reference to Rate Effects	30
1.19	Some Proposed Formulae for Correlating Stress, Strain, Strain Rate and Temperature	32
1.20	The Yield Stress of Steel at about 15 °C	33
	References	34

Contents

CHAPTER 2 STRESS

2.1	Definitions	39
2.2	The Equations of Force Equilibrium	41
2.3	Couple Equilibrium	44
2.4	Three-dimensional Stress Systems	44
2.5	Mohr's Circles for Three-dimensional Stress Systems	49
2.6	The Shortcoming of Mohr's Circle for Three-dimensional Stress Systems	53
	References	54

CHAPTER 3 STRAIN

3.1	Infinitesimal Strains as Functions of Displacement	55
3.2	A Note on the Strain Tensor	59
3.3	The Geometry of Large and Small Strains	59

CHAPTER 4 THE YIELD CRITERIA OF METALS

4.1	General Considerations	63
4.2	Stress Space Representations of Yield Criteria	65
4.3	Shear and Volumetric Resilience and the Mises Criterion	70
4.4	Experimental Evidence for the Tresca and Mises Criteria of Yielding	72
4.5	Isotropic Work-hardening Materials	75
4.6	An Anisotropic Yield Criterion	76
	References	78

CHAPTER 5 STRESS-STRAIN RELATIONS

5.1	The Elastic Stress-Strain Relations	80
5.2	The Prandtl-Reuss Equations	81
5.3	The Lévy-Mises Equations	83
5.4	Work-hardening	83
5.5	The Complete Stress-Strain Relations	86
5.6	Total Strain Theory	87
5.7	The Lévy-Lode Variables	88
5.8	The Plastic Potential and Flow Rules	89
5.9	The Principle of Maximum Work Dissipation	91
5.10	Flow Rule for Anisotropic Material	94
5.11	Generalized Stress and Strain Relations for Anisotropic Work-hardening Material	96
	References	97

CHAPTER 6 METHODS OF DETERMINING WORK-HARDENING CHARACTERISTICS

6.1	Introduction	100
6.2	Simple Tension	102

6.3	Balanced Biaxial Tension	103
6.4	Rolling and Simple Tension	109
6.5	Simple Compression	110
6.6	Plane Strain Compression	114
6.7	Simple Torsion	118
	References	121

CHAPTER 7 ELEMENTARY ANALYSES OF THE ELASTIC-PLASTIC BENDING OF BEAMS, RINGS AND PLATES

7.1	Introduction	124
7.2	Simple Theory of Plastic Bending	125
7.2.1	Straight Rectangular Section Beams	125
7.2.2	Expressions for M , Using a Non-linear Stress-Strain Law	127
7.2.3	Expression for Deflection	127
7.2.4	Shear Stress Distribution	128
7.2.5	Idealized Materials in Bending	129
7.2.6	Shape Factor	131
7.2.7	Plastic Asymmetrical Bending	133
7.3	Plastic Bending Followed by Elastic Unloading	133
7.3.1	Residual Stress Distribution	133
7.3.2	Springback Calculations	135
7.4	The Collapse Load in Simple Structures: Plastic Hinges	137
7.4.1	The Built-in Beam	137
7.4.2	Portal Frames	138
7.4.3	Oval Links and Circular Rings	139
7.4.4	Stud Link	141
7.5	The Diagram of Angular Velocities	142
7.6	The Plane Strain Bending of Cantilevers by a Transverse Shear Force	144
7.7	Bending in Wide Ring Cogging	145
7.8	Combined Bending and Tension: Example of Use of a Yield Inequality	146
7.8.1	Elastic Analysis	146
7.8.2	Plastic Analysis	147
7.9	The Elastic-Plastic Bending of Wide Plates having an Initial Curvature	150
7.9.1	Purely Elastic Case	151
7.9.2	Fully Plastic Case	151
7.9.3	Elastic-Plastic Case	152
7.9.4	Residual Stresses	153
7.9.5	Additional References	154
7.10	Sheet and Plate Bending as a Forming Operation	154
7.10.1	Press Brake Forming of Straight Edges	154
7.10.2	Plate Bending Using Bending Machines	155
	References	156

CHAPTER 8 TORSION OF PRISMATIC BARS OF CIRCULAR AND NON-CIRCULAR SECTION		
8.1	Elastic Analyses	159
8.1.1	Introduction	159
8.1.2	The Analysis of Torsion Following Saint-Venant	160
8.1.3	Elliptical Cross-section Bar	165
8.1.4	Equilateral Triangle	167
8.1.5	The Straightforward Nature of the above two Examples	167
8.1.6	Rectangular and Square Sections	169
8.1.7	A Circular Shaft with a Circular Keyway	170
8.1.8	The Semi-inverse Solution to Saint-Venant's Torsion Problem	171
8.1.9	Point Matching	171
8.1.10	The Torsion of Hollow Cylinders	173
8.1.11	The Membrane Analogy	175
8.1.12	Other Analogies	182
8.2	Plastic Analyses	183
8.2.1	Plastic Yielding in a Prismatic Bar and the Sand Heap Analogy	183
8.2.2	Unloading in Hollow Bars Following Elastic-Plastic Monotonic Torsion	190
8.2.3	Elastic-Plastic Torsion with Work-hardening	190
8.3	Residual Stresses in Plastically Twisted Shafts	190
8.3.1	Circular Shafts	190
8.3.2	The Equilateral Triangle	192
8.3.3	Using the Sand Heap and Membrane Analogies	193
8.4	The Elastic Shortening of Twisted Bars	195
8.5	The Plastic-Torsion of I-Sections with Warping Restraint	197
	References	198
CHAPTER 9 ELASTIC-PLASTIC PROBLEMS WITH SPHERICAL OR CYLINDRICAL SYMMETRY		
9.1	Introduction	201
9.2	Thick Hollow Spheres	202
9.2.1	Elastic Stress Distribution: Steady State Temperature Gradient Only	202
9.2.2	Elastic Stress Distribution: Internal Pressure, p , Only	204
9.2.3	Elastic Stress Distribution: Internal Pressure and a Steady State Temperature Gradient: Onset of Yielding	204
9.2.4	Partly Plastic Shell: Internal Pressure Only	209
9.2.5	Partly Plastic Shell: Temperature Gradient Only	210
9.2.6	Partly Plastic Shell: Steady State Radial Temperature Gradient and an Internal or External Pressure	213
9.2.7	The Influence of Displacements: Internal Pressure Only	214
9.2.8	Expanding an Infinitely Small Cavity by Internal Pressure	216
9.2.9	The Work-Hardening Material	216

9.2.10	The Metallic Spherical Shell: Other References	218
9.2.11	Stress in a Sphere Due to a Uniform Heat Source, Q per Unit Volume	218
9.3	Thick Circular Cylinders: Plane Strain	219
9.3.1	Elastic Stress Distribution: General Equations	219
9.3.2	Elastic Stress Distribution: Any Steady State Temperature Distribution	221
9.3.3	Elastic Stress Distribution: Steady State Heat Flow: Constant Temperature Difference between the Cylinder Walls Only	222
9.3.4	Elastic Stress Distribution: Internal Pressure Only	222
9.3.5	Elastic Stress Distribution: Steady State Temperature Gradient and Internal Pressure	223
9.3.6	Brittle Failure	223
9.3.7	Elastic-Plastic Tube: Internal Pressure Only	223
9.3.8	Onset of Ductile Yielding: Internal Pressure and Temperature Gradient	225
9.3.9	Determination of the Pressure-Expansion Curve in a Thick-walled Tube which is closed at its ends	225
9.3.10	Elastic Thermal Shock	228
9.3.11	Special References	228
9.4	Compound Circular Cylinders and Spherical Shells	229
9.4.1	The Compound Cylinder Subjected to Internal Pressure Only	229
9.4.2	Spherical Shells	233
9.5	Rings Subject to a Steady State Radial Temperature Gradient Only	233
9.6	Rotating Discs	234
9.6.1	Elastic Analysis	234
9.6.2	Elastic-Plastic Analysis	236
9.7	Elastic-Plastic Analysis of a Rotating Cylinder	238
	References	239

CHAPTER 10 PLASTIC INSTABILITY

10.1	General Considerations of the Buckling of an Ideal Column	243
10.2	The Double-modulus Formula	246
10.3	The Tangent-modulus Formula	248
10.4	A Comparison Between the two Solutions for the Plastic Buckling of a Column with a Rectangular Cross-section	248
10.5	General Considerations of Plastic Instability in Tension	249
10.6	Plastic Instability of a Closed-ended Thin-walled Pipe or Cylinder Subjected to Internal Pressure	252
10.7	Plastic Instability of a Spherical Shell Subjected to Internal Pressure	256
10.8	Plastic Instability of a Thin-walled Pipe or Cylinder Subjected to an Internal Pressure and Independent Axial Load	257

10.9	Instability of a Circular Metal Diaphragm	259
10.10	The Strength of Thin-walled Shells and Circular Diaphragms subjected to Hydrostatic Pressure	264
10.10.1	Thin-walled Pipe or Cylinder	264
10.10.2	Spherical Shell	266
10.10.3	Circular Diaphragm	266
10.11	Empirical Equations to represent the Stress-Strain Curve	267
10.12	Instability in Rotating Discs	270
10.12.1	Disc of Uniform Strength	270
10.12.2	Disc of Uniform Thickness	276
	References	278

CHAPTER 11 MECHANICS OF METAL FORMING I

11.1	Introduction	281
11.2	The Sinking of a Thin-walled Tube: General Considerations	281
11.3	Frictionless Tube-sinking	285
11.4	Tube-sinking with Wall Friction	286
11.5	Comparison of Theoretical and Experimental Stresses in Tube-sinking	288
11.6	The Detection and Measurement of Residual Stresses in Cold-drawn Tubes	290
11.7	Deep-drawing of a Circular Blank	292
11.7.1	General Considerations	292
11.7.2	Pure Radial Drawing	301
11.8	Ironing	306
11.9	The Re-drawing of Cups	307
11.10	Flange Wrinkling in Deep-drawing	311
11.11	Blanking	316
11.12	Wire-drawing	321
11.12.1	Theories and Experiment	321
11.12.2	Redundant Strain and Redundant Work Factors in Drawn Strain-hardening Materials	328
11.12.3	The Maximum Reduction in Drawing: A Simple Analysis	329
11.13	Extrusion: General Considerations	330
11.14	Determination of Extrusion Pressure	334
11.15	Rolling	338
11.15.1	Cold-rolling of Strip	338
11.15.2	Rolling Thin Hard Strip	346
11.15.3	Ring Rolling	346
11.15.4	Asymmetrical Rolling	347
11.15.5	The Pendulum Mill	348
11.15.6	Hot-rolling of Strip	348
11.15.7	Section-rolling, Transverse Rolling, V-Groove Forming and Spread	349
11.15.8	Seamless Tube Making	349
11.16	Swaging a Cylindrical Rod	350

11.17	The Simple Upsetting or Compressing of a Cylinder	350
11.18	The Compression of Non-circular Prismatic Blocks	356
11.19	Some Relationships between Engineering Strain Rate, Force, Time and Strain, in a Simple Upsetting Operation which takes Place under a Drop Hammer	358
11.20	Superplasticity	367
	11.20.1 Transformation Plasticity	368
	11.20.2 Micrograin Plasticity	368
	References	370

CHAPTER 12 THE SLIP-LINE FIELD: THEORY AND EXAMPLES OF PLANE PLASTIC STRAIN

12.1	General Remarks	381
12.2	The Stress Equations	383
12.3	The Velocity Equations	387
12.4	A Simple Slip-line Field for Extrusion through a Frictionless Wedge-shaped Die of Semi-angle α , of Reduction $r = 2 \sin \alpha / (1 + 2 \sin \alpha)$	389
12.5	The Compression of a Block between Rough Rigid Parallel Platens, the Platen Width Exceeding the Material Thickness	392
12.6	The Centred-fan Field	401
12.7	Examples Using the Centred-fan Field	402
	12.7.1 Extrusion through a Square Die over a Smooth Container Wall; $r > 0.5$	402
	12.7.2 Extrusion through a Smooth Wedge-shaped Die of Semi-angle α when $r \geq 2 \sin \alpha / (1 + 2 \sin \alpha)$	403
	12.7.3 Extrusion through a Smooth Wedge-shaped Die of Semi-angle α where $r \leq 2 \sin \alpha / (1 + 2 \sin \alpha)$	403
	12.7.4 Simultaneous Extrusion through Two Orifices in a Square Die	406
	12.7.5 An Extrusion Involving Rotation	407
12.8	Stubby Cantilevers and Beams Carrying a Concentrated Load	408
12.9	Concluding Note	412
	References	414

CHAPTER 13 LOAD BOUNDING: INTRODUCTION AND APPLICATION TO PLANE STRAIN DEFORMATION PROBLEMS

13.1	Introduction	415
	13.1.1 The Limit Theorems	416
13.2	The Lower Bound Theorem	416
13.3	The Upper Bound Theorem	418
13.4	The Upper Bound Theorem in Plane Strain: Elementary Justification	419
13.5	Examples of the Use of the Upper Bound Theorem	421
	13.5.1 Bending of a Notched Bar	422

13.5.2	Simple Indentation	423
13.5.3	Compression Between Smooth Plates: Heat Lines	424
13.5.4	Extrusion through Symmetrical Wedge-shaped Dies	426
13.5.5	Extrusion through Unsymmetrical Wedge-shaped Dies	429
13.5.6	Extrusion through Square Dies	431
13.5.7	End Extrusion through Three Holes	434
13.5.8	Sideways Extrusion from a Smooth Container	434
13.5.9	Simultaneous Forward and Backward Extrusion of Sheets of Equal Thickness	435
13.5.10	Simultaneous Piercing by Opposed Punches	436
13.6	Friction	438
13.7	Closed Die Forging	439
13.8	Extrusion through Curved Dies	439
13.9	Rolling	443
13.10	Temperature Distribution in Swiftly Worked Metals	445
13.11	Temperature Distribution in Extrusion Due to Fast Working	447
13.12	A Simple Notation for Constructing Hodographs	449
13.13	Examples of the Application of the Lower Bound Theorem Using Stress Discontinuity Patterns	451
13.13.1	The Mohr Circle in Plane Strain: The Pole of the Mohr Circle	451
13.13.2	Stress Discontinuities, or Jumps, in Plane Strain	452
13.13.3	The Notched Bar in Bending	453
13.13.4	The Tapering Cantilever Under Shear Loading	454
13.13.5	Sheet Drawing and Sheet Extrusion for Frictionless Wedge-shaped Dies	456
13.13.6	Indentation	458
13.13.7	Large Reduction Extrusion through Square Dies	459
13.13.8	Hot-rolling	461
13.14	An Analogy: Minimum Weight Frames: Michell Structures	461
13.15	Upper Bounds for Anisotropic Metals	463
	References	464

CHAPTER 14 MECHANICS OF METAL FORMING II

14.1	Introduction	467
14.2	Machining	467
14.2.1	General	467
14.2.2	Ernst and Merchant's Theory	469
14.2.3	The Theory of Lee and Shaffer	473
14.2.4	Remarks on the Comparison Between Theoretical and Experimental Results	474
14.2.5	Bibliographical Note	475
14.2.6	Discontinuous Machining	477
14.2.7	Oblique Machining	478
14.2.8	Machining Using a Restricted Contact Tool	478
14.2.9	Turning and Boring: Some Simple Upper Bounds	489
14.3	Indenting with Straight-sided Dies	494

14.4	Indenting and Forging with a Flat Punch	497
14.5	Opposed Indenters: Forging and Cutting	503
14.6	Slip-Line Fields for Hot- and Cold-rolling	504
14.7	Strip and Bar Drawing through a Perfect Die	505
14.8	Extrusion through Bi-wedge Shaped and Curved Dies	508
14.9	Rotating Dies and Elements	510
14.10	Plastic Deformation of Metals of Different Yield Strengths	513
	References	514

CHAPTER 15 LOAD BOUNDING APPLIED TO THE PLASTIC BENDING OF PLATES

15.1	Introduction	520
15.2	An Annular Plate Clamped at its Outer Edge	521
15.3	Plate Position Fixed (Zero Fixing Moment) at its Outer Periphery	522
15.4	Plate Perfectly Clamped along its Inner Boundary	523
15.5	The Dynamic, or Inertia Loading of an Annular Plate Perfectly Clamped along its Inner Boundary	524
15.6	Regular Polygonal Plates	525
15.7	Rectangular Plate: Uniform Loading	526
15.8	The Position-fixed Rectangular Plate Carrying a Concentrated Load	528
15.9	A Plate Shape which is an Equilateral Triangle	530
15.9.1	The Concentrated Load	530
15.9.2	The Uniformly Distributed Load	532
15.10	Simply Supported Plates	532
15.11	A Square Plate Supported only at its Corners	534
15.12	A Position-fixed Rectangular Plate, Unsymmetrically Loaded by a Concentrated Load	535
15.13	Local Collapse in a Triangular Plate	538
15.14	A Plate Twisting Analogy	540
15.15	The Elastic-Plastic Bending of Circular Plates by Transverse Pressure	541
	References	543

CHAPTER 16 LOAD BOUNDING APPLIED TO AXISYMMETRIC INDENTATION AND RELATED PROBLEMS

16.1	Introduction	544
16.2	Basic Equations	544
16.3	Simple Heading Operations with Frictionless Tools	545
16.3.1	Relatively Narrow Bands: First Mode of Deformation	545
16.3.2	Wide Bands: Second Mode of Deformation	547
16.3.3	A Third Mode of Deformation for a Very Deep Band	547
16.4	Three-dimensional Punch Indentation Problem	547
16.4.1	Lower Bound for a Rectangular Punch	548
16.4.2	Upper Bound for a Rectangular Punch	548

16.4.3	Indentation with a Pyramid Indenter	550
16.4.4	Comment	551
16.5	An Upper Bound for the Load to Compress Square Discs between Rough, Parallel, Rigid, Overhanging Dies	551
16.5.1	Simple, Homogeneous Velocity Field	551
16.5.2	Velocity Field which Allows Bulging at Edges of Block	553
16.6	The Indentation of a Thin Cylindrical Slab by a Pair of Rough Rigid Punches	554
16.6.1	First Mode of Deformation: Disc Wholly Plastic	555
16.6.2	Second Mode of Deformation: Outer Annulus of Disc Remaining Rigid	556
16.7	Velocity Fields for Axisymmetric Extrusion, Rolling and Indentation	557
16.8	Hill's General Method	559
	References	559
	PROBLEMS	561
	SOLUTIONS TO PROBLEMS	586
	AUTHOR INDEX	635
	SUBJECT INDEX	643