
Contents

<i>Preface</i>	<i>xi</i>
<i>About the front cover</i>	<i>xiii</i>
<i>Symbols and abbreviations</i>	<i>xv</i>
1 Introduction to dynamic systems	1
1.1 <i>What is dynamics and why is it important?</i>	1
1.1.1 <i>Why dynamics matters</i>	2
1.1.1.1 <i>The Tacoma Narrows Bridge</i>	2
1.1.1.2 <i>The Northridge and Kobe Earthquakes</i>	3
1.1.1.3 <i>The London Millennium Bridge</i>	5
1.2 <i>Basic definitions</i>	6
1.3 <i>Describing oscillatory motion</i>	7
1.3.1 <i>Time and frequency domains</i>	7
1.3.2 <i>The basics of harmonic motion</i>	8
1.3.3 <i>Phasors</i>	12
1.3.4 <i>Some everyday examples of harmonic motion</i>	13
1.3.4.1 <i>Pendulum</i>	13
1.3.4.2 <i>Bobbing in water</i>	13
1.4 <i>Dynamic properties of physical systems</i>	14
1.4.1 <i>Mass and stiffness</i>	14
1.4.2 <i>Damping</i>	15
1.4.3 <i>Modelling physical systems for dynamic analysis</i>	17
1.5 <i>Dynamic loads</i>	19
1.5.1 <i>Impulsive loads</i>	19
1.5.2 <i>Periodic loads</i>	20
1.5.3 <i>Random loads</i>	22
<i>Key points</i>	24
<i>Tutorial problems</i>	24

2	Single-degree-of-freedom systems	27
2.1	Introduction	27
2.2	Equation of motion	27
2.2.1	With an applied force	27
2.2.2	With a ground motion	29
2.3	Free vibration	30
2.3.1	Undamped free vibration	31
2.3.2	Damped free vibration	33
2.3.2.1	Critically damped: $\xi = 1$ (or $c^2 = 4 km$)	34
2.3.2.2	Underdamped: $\xi < 1$ (or $c^2 < 4 km$)	35
2.3.2.3	Overdamped: $\xi > 1$ (or $c^2 > 4 km$)	36
2.3.2.4	Logarithmic decrement	36
2.4	Response to dynamic loads	37
2.4.1	Step load	38
2.4.2	Harmonic load	40
2.4.3	Harmonic base motion	45
2.4.4	Response to periodic loads	49
2.4.5	Impulse response	52
2.4.6	Response to irregular dynamic loads	54
2.4.7	Earthquake response spectrum	59
2.5	Transfer functions and block diagrams	64
2.5.1	Laplace transforms	65
2.5.2	Transfer function of a linear SDOF system	66
2.5.3	Relationship to impulse response	68
	Key points	69
	Tutorial problems	69
3	Multi-degree-of-freedom systems	73
3.1	Introduction	73
3.2	Equations of motion	74
3.2.1	MDOF systems with dynamic forces	74
3.2.2	MDOF systems subjected to base motions	77
3.2.3	Some comments on MDOF equations of motion	79
3.3	Free vibration of MDOF systems	80
3.3.1	Natural frequencies and mode shapes	80
3.3.2	Properties of mode shapes	86
3.3.3	Empirical methods	88
3.4	Vibration analysis of 2DOF systems	89
3.4.1	Equivalence between 2DOF and two SDOF systems	89
3.4.2	Undamped response to a harmonic load	92
3.4.3	Tuned mass dampers	94
3.5	Response of MDOF systems to dynamic loads	96
3.5.1	Mode superposition method: theory	96
3.5.2	Modelling damping in MDOF systems	97

- 3.5.2.1 *Modal damping* 98
- 3.5.2.2 *Rayleigh damping* 98
- 3.5.2.3 *Other damping formulations* 99
- 3.5.3 *Implementing the mode superposition method* 100
- 3.5.4 *Mode superposition for MDOF systems with base motions* 104
- 3.5.5 *Truncation of modes* 106
- 3.5.6 *Mode superposition and response spectrum analysis* 111

Key points 113

Tutorial problems 113

4 Continuous systems 117

- 4.1 *Introduction* 117
- 4.2 *Vibrations of beams* 117
 - 4.2.1 *Undamped equation of motion* 117
 - 4.2.2 *Undamped free vibration of uniform beams* 119
 - 4.2.3 *Forced vibration response* 124
- 4.3 *Equivalent single-degree-of-freedom systems* 128
 - 4.3.1 *Generalised properties* 128
 - 4.3.2 *Response to applied loads* 133
 - 4.3.3 *Response to a base motion* 135

Key points 138

Tutorial problems 139

5 Non-linear dynamics 141

- 5.1 *Introduction* 141
- 5.2 *Non-linearity in structural properties* 142
 - 5.2.1 *Variable mass* 142
 - 5.2.2 *Non-proportional damping* 142
 - 5.2.3 *Non-linear stiffness* 147
 - 5.2.3.1 *Elastic-perfectly plastic hysteresis* 149
 - 5.2.3.2 *A bilinear stiffness model* 150
 - 5.2.4 *Geometric non-linearity* 152
 - 5.2.5 *Dynamic characteristics of non-linear systems* 156
- 5.3 *Time-stepping algorithms* 158
 - 5.3.1 *The need for a different approach* 158
 - 5.3.2 *A simple algorithm: the central difference method* 159
 - 5.3.3 *Stability, accuracy and time step* 165
 - 5.3.3.1 *Accuracy* 165
 - 5.3.3.2 *Stability* 167
 - 5.3.4 *A more complex algorithm: the Newmark method* 169
 - 5.3.5 *Extension to MDOF systems* 171
 - 5.3.5.1 *Damping* 171
 - 5.3.5.2 *Stability, accuracy and time step* 172
 - 5.3.6 *How many non-linear analyses?* 172

5.4	<i>Approximate treatment of non-linearity</i>	174
5.4.1	<i>Ductility-modified spectra</i>	174
5.4.2	<i>Application to strain hardening and MDOF structures</i>	179
	<i>Key points</i>	181
	<i>Tutorial problems</i>	181

6 Fourier analysis and random vibrations 183

6.1	<i>Introduction</i>	183
6.2	<i>Properties of random variables and processes</i>	184
6.2.1	<i>Deterministic and random variables</i>	184
6.2.2	<i>Discrete and continuous random variables</i>	184
6.2.3	<i>Statistical properties of random variables</i>	187
6.2.4	<i>Two random variables</i>	188
6.2.5	<i>Properties of random processes</i>	189
6.3	<i>Autocorrelation</i>	191
6.3.1	<i>Convolution integral</i>	191
6.3.2	<i>Autocorrelation function</i>	192
6.4	<i>Fourier analysis</i>	197
6.4.1	<i>Fourier transform</i>	197
6.4.2	<i>Properties of Fourier transforms</i>	200
6.4.3	<i>The Dirac delta function</i>	200
6.4.4	<i>Practical use of Fourier transforms</i>	202
6.5	<i>Power spectral density</i>	203
6.5.1	<i>White noise</i>	206
6.6	<i>Impulse response and transfer function</i>	208
6.6.1	<i>Impulse response function</i>	208
6.6.2	<i>Transfer function</i>	211
6.6.3	<i>Application to random vibration analysis</i>	212
6.6.4	<i>Modal testing</i>	214
	<i>Key points</i>	215
	<i>Tutorial problems</i>	216

APPENDICES

Background mathematics	219
<i>Appendix A: Basic trigonometry and the complex exponential</i>	221
<i>Appendix B: Second order ordinary differential equations</i>	223
<i>Appendix C: Fourier analysis</i>	229
<i>Appendix D: Laplace transforms</i>	235
<i>Appendix E: Eigenvalues and eigenvectors of matrices</i>	241
<i>Appendix F: Partial differential equations</i>	245
<i>Appendix G: Tutorial problems: answers and hints</i>	249
<i>Further reading</i>	255
<i>Index</i>	259