

CONTENTS

Chapter 1 Background

1	Sets and functions	1
2	Algebra of sets	5
3	Cardinal numbers	9
4	Pseudo-metric spaces	11
5	Limits and continuity	15
6	Uniform limits	16
7	Function spaces	19
8	General topology	21

Chapter 2 Measure—General Theory

9	Additive classes and Borel sets	25
10	Additive set functions	31
11	Outer measures	42
12	Regular outer measures	50
13	Metric outer measures	57

Chapter 3 Measure—Specific Examples

14	Lebesgue-Stieltjes measures	68
*15	Probability	78
*16	Hausdorff measures	83
*17	Haar measure	86
18	Nonmeasurable sets	93

Chapter 4 Measurable Functions

19	Definitions and basic properties	97
20	Operations on measurable functions	102
21	Approximation theorems	106
*22	Random variables	112

Chapter 5 Integration

23	The integral of a simple function	116
24	Integrable functions	120
25	Elementary properties of the integral	127
26	Additivity of the integral	132
27	Absolute continuity	136
28	Dominated convergence	145

29	Fubini's theorem	149
*30	Expectation of a random variable	158
Chapter 6 Differentiation		
31	Summary of the problem	167
32	Vitali coverings	171
33	Differentiation of additive set functions	176
34	The Lebesgue decomposition	180
*35	Metric density and approximate continuity	184
*36	Differentiation with respect to nets	191
Chapter 7 Convergence Theorems		
37	Uniform and almost everywhere convergence	199
38	Convergence in measure and in mean	200
39	Relations among convergence types	206
40	Convergence of measures and integrals	208
41	The L_p spaces	215
42	Cauchy theorems	220
*43	Orthogonal expansions in Hilbert space	224
Chapter 8 Functional Analysis		
44	Banach spaces	232
45	The Hahn-Banach theorem	238
46	Representation of linear functionals	245
47	Hamel bases	254
48	Weak and weak* sequential convergence	257
49	Weak* topologies	262
50	The closed-graph theorem	268
Bibliography		
279		
Index of postulates		
281		
Index of symbols		
283		
Subject index		
287		