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The finite element method is a powerful means of analysis that can be traced back to the early 1940s. It can be used to handle any shell problem to any desired degree of accuracy and was early on used to model the behaviour of the basic building elements working up a structure (rather than the behaviour of the whole structure), and then assemble the relevant structural properties by enforcing compatibility conditions between adjacent elements, conditions of equilibrium of forces and moments, as well as boundary conditions at the edges or supports of the structure. In this way, the overall behaviour of the complete structure is approximated. The finite difference method is another popular numerical approach, where the exact differential equations of the problem are approximated with overlapping finite-difference equations, which can then be solved more conveniently as banded algebraic equations. Numerical modelling is particularly useful in those situations where the mathematical formulation results in one or other mathematical problems that are difficult or impossible to solve analytically, particularly where the behaviour is non-linear and/or geometric nonlinearities have to be considered.

However, for situations where the mathematical formulation results in sets of differential equations that can be solved analytically, the numerical solution approach, while often effective as a way of obtaining actual values of stresses and displacements in a structure, is not necessarily the most efficient for the purposes of checking the effect of varying a single parameter (such as the thickness of a shell) during the course of design, since each time such a change is made, the whole analysis procedure has to be carried out, albeit by a computer. Sometimes even the computer program may be necessary in order to obtain a comprehensive understanding of the effect (on the stresses and deformations) of the effect of varying a parameter of interest through the full range of possibilities. Thus the process of engineering design can easily become very expensive if several structural parameters need to be studied.

It is in situations such as these that analytical formulations can be extremely useful. In fact, we may verify the results of exact mathematical solutions to the differential equations of the shell, or at least where numerical solution approaches have become very popular (owing to their ease of programming), on the grounds that analytical solutions (or provide valuable insight) on the key parameters that govern the behaviour of the shell, or are more suitable as systematic parametric study than numerical formulations and do provide a means (sometimes the only means) for checking the correctness of numerical solution procedures.