

# Contents

<i>Preface to Volume 3</i>	xiii	
1	Introduction and the equations of fluid dynamics	1
1.1	General remarks and classification of fluid mechanics problems discussed in this book	1
1.2	The governing equations of fluid dynamics	4
1.3	Incompressible (or nearly incompressible) flows	10
1.4	Concluding remarks	12
	References	12
2	Convection dominated problems – finite element approximations to the convection–diffusion equation	13
2.1	Introduction	13
2.2	The steady-state problem in one dimension	15
2.3	The steady-state problem in two (or three) dimensions	26
2.4	Steady state – concluding remarks	30
2.5	Transients – introductory remarks	32
2.6	Characteristic-based methods	35
2.7	Taylor–Galerkin procedures for scalar variables	47
2.8	Steady-state condition	48
2.9	Non-linear waves and shocks	48
2.10	Vector-valued variables	52
2.11	Summary and concluding remarks	59
	References	59
3	A general algorithm for compressible and incompressible flows – the characteristic-based split (CBS) algorithm	64
3.1	Introduction	64
3.2	Characteristic-based split (CBS) algorithm	67
3.3	Explicit, semi-implicit and nearly implicit forms	76
3.4	‘Circumventing’ the Babuška–Brezzi (BB) restrictions	78
3.5	A single-step version	80
3.6	Boundary conditions	81

3.7	The performance of two- and single-step algorithms on an inviscid problem	85
3.8	Concluding remarks	87
	References	87
4	Incompressible laminar flow – newtonian and non-newtonian fluids	91
4.1	Introduction and the basic equations	91
4.2	Inviscid, incompressible flow (potential flow)	93
4.3	Use of the CBS algorithm for incompressible or nearly incompressible flows	97
4.4	Boundary-exit conditions	100
4.5	Adaptive mesh refinement	102
4.6	Adaptive mesh generation for transient problems	113
4.7	Importance of stabilizing convective terms	113
4.8	Slow flows – mixed and penalty formulations	113
4.9	Non-newtonian flows – metal and polymer forming	118
4.10	Direct displacement approach to transient metal forming	132
4.11	Concluding remarks	133
	References	134
5	Free surfaces, buoyancy and turbulent incompressible flows	143
5.1	Introduction	143
5.2	Free surface flows	144
5.3	Buoyancy driven flows	153
5.4	Turbulent flows	161
	References	165
6	Compressible high-speed gas flow	169
6.1	Introduction	169
6.2	The governing equations	170
6.3	Boundary conditions – subsonic and supersonic flow	171
6.4	Numerical approximations and the CBS algorithm	173
6.5	Shock capture	174
6.6	Some preliminary examples for the Euler equation	176
6.7	Adaptive refinement and shock capture in Euler problems	180
6.8	Three-dimensional inviscid examples in steady state	188
6.9	Transient two and three-dimensional problems	195
6.10	Viscous problems in two dimensions	197
6.11	Three-dimensional viscous problems	207
6.12	Boundary layer–inviscid Euler solution coupling	209
6.13	Concluding remarks	212
	References	212
7	Shallow-water problems	218
7.1	Introduction	218
7.2	The basis of the shallow-water equations	219
7.3	Numerical approximation	223

7.4	Examples of application	224
7.5	Drying areas	236
7.6	Shallow-water transport	237
	References	239
8	Waves	242
8.1	Introduction and equations	242
8.2	Waves in closed domains – finite element models	243
8.3	Difficulties in modelling surface waves	245
8.4	Bed friction and other effects	245
8.5	The short-wave problem	245
8.6	Waves in unbounded domains (exterior surface wave problems)	250
8.7	Unbounded problems	253
8.8	Boundary dampers	253
8.9	Linking to exterior solutions	255
8.10	Infinite elements	259
8.11	Mapped periodic infinite elements	260
8.12	Ellipsoidal type infinite elements of Burnett and Holford	261
8.13	Wave envelope infinite elements	262
8.14	Accuracy of infinite elements	264
8.15	Transient problems	265
8.16	Three-dimensional effects in surface waves	266
	References	270
9	Computer implementation of the CBS algorithm	274
9.1	Introduction	274
9.2	The data input module	275
9.3	Solution module	278
9.4	Output module	289
9.5	Possible extensions to CBSflow	289
	References	289
	Appendix A: Non-conservative form of Navier–Stokes equations	291
	Appendix B: Discontinuous Galerkin methods in the solution of the convection–diffusion equation	293
	Appendix C: Edge-based finite element formulation	298
	Appendix D: Multigrid methods	300
	Appendix E: Boundary layer–inviscid flow coupling	302
	Author index	307
	Subject index	315