

# Table of Contents

|                                                                          |    |
|--------------------------------------------------------------------------|----|
| <b>Preface</b> .....                                                     | ix |
| <b>Chapter 1. Introduction</b>                                           |    |
| 1. The direct method .....                                               | 1  |
| 2. Parametrized measures: some examples .....                            | 3  |
| 3. A general framework .....                                             | 8  |
| 4. The case curl: gradient parametrized measures .....                   | 13 |
| 5. Some general facts about parametrized measures .....                  | 20 |
| 6. Appendix .....                                                        | 21 |
| 7. Bibliographical remarks .....                                         | 23 |
| <b>Chapter 2. Some Variational Problems</b>                              |    |
| 1. Introduction .....                                                    | 25 |
| 2. Variational problems under no differential constraint .....           | 26 |
| 3. Optimal control problems .....                                        | 29 |
| 4. An optimal design problem .....                                       | 32 |
| 5. Turbulent fluids .....                                                | 37 |
| 6. Bibliographical remarks .....                                         | 41 |
| <b>Chapter 3. The Calculus of Variations under Convexity Assumptions</b> |    |
| 1. Introduction .....                                                    | 43 |
| 2. Weak lower semicontinuity .....                                       | 44 |
| 3. Existence theorems .....                                              | 49 |
| 4. Polyconvexity .....                                                   | 51 |
| 5. A brief account of non-linear elasticity .....                        | 54 |
| 6. Weak and strong convergence of minimizing sequences .....             | 56 |
| 7. Bibliographical remarks .....                                         | 59 |
| <b>Chapter 4. Nonconvexity and Relaxation</b>                            |    |
| 1. Introduction .....                                                    | 61 |
| 2. Relaxation theorem .....                                              | 63 |
| 3. Parametrized measures solutions of variational principles .....       | 65 |
| 4. Bibliographical remarks .....                                         | 70 |

**Chapter 5. Phase Transitions and Microstructure**

|                                                     |    |
|-----------------------------------------------------|----|
| 1. Two main examples from continuum mechanics ..... | 71 |
| 2. Phase transitions and microstructure .....       | 74 |
| 3. The two-well problem .....                       | 76 |
| 4. An example in micromagnetics .....               | 89 |
| 5. Bibliographical remarks .....                    | 94 |

**Chapter 6. Parametrized Measures**

|                                                                  |     |
|------------------------------------------------------------------|-----|
| 1. Introduction .....                                            | 95  |
| 2. Existence theorem .....                                       | 96  |
| 3. Sufficient conditions for representation of weak limits ..... | 102 |
| 4. Chacon's biting lemma and biting convergence .....            | 105 |
| 5. Biting convergence and weak convergence .....                 | 108 |
| 6. Strong convergence .....                                      | 111 |
| 7. Appendix .....                                                | 112 |
| 8. Bibliographical remarks .....                                 | 113 |

**Chapter 7. Analysis of Parametrized Measures**

|                                                    |     |
|----------------------------------------------------|-----|
| 1. Introduction .....                              | 115 |
| 2. Homogenization and localization .....           | 117 |
| 3. Riemann-Lebesgue lemma .....                    | 120 |
| 4. Two auxiliary lemmas .....                      | 122 |
| 5. The homogeneous case .....                      | 124 |
| 6. Characterization of parametrized measures ..... | 126 |
| 7. Appendix .....                                  | 128 |
| 8. Bibliographical remarks .....                   | 130 |

**Chapter 8. Analysis of Gradient Parametrized Measures**

|                                                                   |     |
|-------------------------------------------------------------------|-----|
| 1. Introduction .....                                             | 133 |
| 2. Homogenization, localization and the Riemann-Lebesgue lemma .. | 135 |
| 3. The scalar case .....                                          | 139 |
| 4. Quasiconvexity .....                                           | 143 |
| 5. $W^{1,p}$ -quasiconvexity .....                                | 149 |
| 6. The vector case: proof of necessity .....                      | 150 |
| 7. The vector case: proof of sufficiency .....                    | 153 |
| 8. Appendix .....                                                 | 157 |
| 9. Bibliographical remarks .....                                  | 158 |

**Chapter 9. Quasiconvexity and Rank-one Convexity**

|                                                           |     |
|-----------------------------------------------------------|-----|
| 1. Introduction .....                                     | 161 |
| 2. Laminates .....                                        | 162 |
| 3. Characterization of laminates .....                    | 166 |
| 4. A microstructure that is not a laminate .....          | 168 |
| 5. Rank-one convexity does not imply quasiconvexity ..... | 173 |
| 6. Bibliographical remarks .....                          | 177 |

**Chapter 10. Analysis of Divergence-Free Parametrized Measures**

|                                                                    |     |
|--------------------------------------------------------------------|-----|
| 1. Introduction .....                                              | 179 |
| 2. Technical preliminaries .....                                   | 182 |
| 3. Convexification .....                                           | 185 |
| 4. Characterization of divergence-free parametrized measures ..... | 186 |
| 5. Bibliographical remarks .....                                   | 191 |

|                           |     |
|---------------------------|-----|
| <b>Bibliography</b> ..... | 193 |
|---------------------------|-----|

|                    |     |
|--------------------|-----|
| <b>Index</b> ..... | 211 |
|--------------------|-----|