
Contents

Part I	Theory and Code Development	1
1	Boundary Value Problems	3
1.1	Single Elliptic Equation	3
1.1.1	Classical Formulation	3
1.1.2	Variational Formulation	4
1.1.3	Linear Acoustics Equations	5
1.1.4	Axisymmetric Elliptic Problems	8
1.2	Linear Elasticity	9
1.2.1	Elasticity in Cylindrical Coordinates: Axisymmetric Problems	11
1.3	Maxwell Equations	13
1.3.1	Axisymmetric Maxwell Problems	16
1.4	Elasticity Coupled with Acoustics	18
2	Exact hp Sequences, Projection-Based Interpolation, De Rham Diagrams	25
2.1	Exact Polynomial Sequences	26
2.1.1	Nédélec's Tetrahedron of the Second Type	27
2.1.2	Nédélec's Hexahedron of the First Type	29
2.1.3	Nédélec's Tetrahedron of the First Type	31
2.1.4	Prismatic Elements	37
2.1.5	Parametric Elements	37
2.1.6	Pyramid Elements	39
2.2	H^1 -, $H(\text{curl})$ -, and $H(\text{div})$ -Conforming Projection-Based Interpolation	44
2.2.1	Commuting Projection-Based Interpolation in 1D	44
2.2.2	Commuting Projection-Based Interpolation in 2D	46
2.2.3	Commuting Projection-Based Interpolation in 3D	47
2.2.4	Discussion	48
2.3	Shape Functions	50
3	3D hp Finite Element Method	61
3.1	Construction of FE Basis Functions on Regular Meshes	61
3.2	Supported h -Refinements	65
3.2.1	h -Refinement Algorithm	66
3.2.2	1-Irregular Meshes Algorithm	68
3.2.3	Modified 1-Irregular Meshes Algorithm	72

3.3	p -Refinements and the Minimum Rule	74
3.4	Constrained Approximation	75
3.4.1	Nodes Constrained by an Edge	75
3.4.2	Nodes Constrained by a Face	76
3.4.3	Modified Element	78
4	3 Dhp Code	81
4.1	Organization of the 3 Dhp Code	81
4.2	Data Structure in FORTRAN 90	82
4.3	Data Structure Supporting Algorithms	84
4.3.1	Natural Order of Elements	84
4.3.2	Determining Middle Node Neighbors for a Mid-Face Node	86
4.3.3	Determining Middle Node Neighbors for a Mid-Edge Node	87
4.3.4	Reconstructing Nodal Connectivities	88
5	Geometry Modeling	91
5.1	GMP Manifold: Compatible Parametrizations	92
5.2	Transfinite Interpolation	94
5.2.1	Transfinite Interpolation for a Hexahedron	94
5.2.2	Transfinite Interpolation for a Rectangle Conforming to a Surface	96
5.3	Interfacing with CUBIT	97
5.4	Exact Geometry and Parametric Elements: Mesh Generation ...	101
5.4.1	How to Define the FE Error	102
6	Automatic hp-Adaptivity in Three Space Dimensions	105
6.1	The hp Algorithm	105
6.1.1	Overview	106
6.1.2	The Edge Refinement Algorithm	107
6.1.3	The Face Refinement Algorithm	110
6.1.4	The Brick Refinement Algorithm	113
6.1.5	Mesh Reconciliation	113
6.1.6	Abstract Framework for Computing Projections	115
6.1.7	The hp Algorithm in $H(\text{curl})$	117
6.2	Goal Oriented hp -Adaptivity	117
6.2.1	Dual Problem and Error Representation Formula	118
6.2.2	The Goal Oriented hp -Algorithm	120
6.3	Examples	122
6.3.1	Fichera's Corner	122
6.3.2	A Manufactured Solution with a Shock	124
6.3.3	Electromagnetic Scattering in a Waveguide	128

7	High-Performance Computation	135
7.1	Fast Integration Algorithm	135
7.2	Telescopic Solver	140
7.2.1	The Symmetric Positive-Definite Case	140
7.2.2	The Symmetric Indefinite Case	143
7.3	Linear Solvers	146
7.3.1	The Symmetric Positive-Definite Case	147
7.3.2	A Domain Decomposition Solver for Acoustics and Electromagnetics	149
8	Two-Grid <i>hp</i> Solver	153
8.1	Formulation	154
8.1.1	Overlapping Block-Jacobi Smoother	154
8.1.1.1	Block-Jacobi Smoother for Electromagnetic Problems	156
8.1.1.2	Specially Designed Blocks for Elongated Elements	158
8.1.2	Coarse-Grid Correction	159
8.1.3	Krylov-Subspace Optimization Methods	160
8.1.4	The Two-Grid Algorithm	163
8.1.5	Stopping Criterion	164
8.2	Elementary Convergence Theory	164
8.2.1	Elliptic Symmetric and Positive-Definite Problems	164
8.2.1.1	Convergence Properties of Smoothing Iterations	164
8.2.1.2	Convergence Properties of Two-Grid Solver Iterations	165
8.2.2	Electromagnetic Problem	166
8.2.2.1	Equivalence between Electromagnetics and an Auxiliary Symmetric and Positive- Definite Problem	168
8.2.2.2	Convergence for Symmetric and Positive- Definite Auxiliary Problem	170
8.3	Implementation Details	172
8.3.1	Block-Jacobi Smoother	172
8.3.2	Stiffness Matrix	173
8.3.3	Coarse Grid Correction and Prolongation/Restriction Operators	173
8.3.4	Embedding Gradients of H^1 into $H(\text{curl})$ for Electromagnetic Problems	174
8.3.5	Krylov Subspace Optimization Methods	175
8.4	Numerical Examples	176
8.4.1	Model Problems	178
8.4.2	Goal-Oriented Two-Grid Solver	178

8.4.3	Elongated Elements	182
8.4.4	Two-Grid Solver in hp -Refined Grids	185
9	A Domain Decomposition Based Parallel Implementation	187
9.1	Mesh Repartitioning: Interfacing with Zoltan	187
9.2	A Nested-Dissections Parallel Multi-Frontal Solver	196
9.3	Parallel Mesh Refinements and Mesh Reconciliation	202
9.4	Numerical Examples	209
Part II	Applications	219
10	Acoustic Scattering Problems	221
10.1	Infinite Element	222
10.2	Examples	225
10.2.1	Acoustic Scattering from Sphere	225
10.2.2	Acoustic Scattering from Cone-Sphere	232
10.2.3	Acoustic Scattering from Thin Square Plate	234
11	Electromagnetic Scattering Problems	237
11.1	Formulation of Scattering Problems	239
11.2	EM Infinite Element	244
11.3	A Domain Decomposition Approach	247
11.3.1	Scattering on a PEC or Dielectric Obstacle	248
11.3.1.1	Solution of Coupled Problem in Terms of Traces	251
11.3.2	Exploiting Symmetries	252
11.3.3	Cavity Backed Aperture in a Flat Infinite Screen: Scattering in $\mathbb{R}_+^3$	255
11.4	Calculation of Radar Cross Section	257
11.4.1	Scattering in $\mathbb{R}^3$ by an Immersed Body Ω	258
11.4.2	Scattering in $\mathbb{R}_+^3$ by a Cavity Backed Aperture in a PEC Ground Plane	260
11.4.3	Scattering in $\mathbb{R}_+^3$ by a Cavity Backed Aperture in an Absorbing Ground Plane	261
11.4.4	Scattered Far-Field Representation	262
11.5	Adaptivity	263
11.5.1	Implicit <i>a Posteriori</i> Error Estimate	263
11.5.2	Explicit Residual Error Estimate	269
11.5.3	Goal-Oriented Error Estimate	271
11.5.4	Goal-Oriented h -Adaptive Strategy	273
11.6	Examples	276
11.6.1	Scattering in $\mathbb{R}^3$ by Immersed Bodies	276
11.6.2	Scattering in a Half-space $\mathbb{R}_+^3$ on Cavity- Backed Apertures	287

12	Three-Dimensional Elasticity and Thin Walled Structures	307
12.1	Introduction	307
12.2	Classical Shell Theory: Comparison with High p Solutions	308
12.2.1	Membrane-Dominated Solutions	310
12.2.2	Membrane-Dominated Solutions of Rotationally Symmetric Shells	311
12.2.3	Examples of Membrane Solutions	313
12.2.4	Bending-Dominated Solutions of Rotationally Symmetric Shells	317
12.2.5	Solutions of Rotationally Symmetric Shells	321
12.3	Solutions of Complex Thin-Walled Structures	325
12.3.1	Joint of Two Cylindrical Thin-Walled Beams	326
12.3.2	Silo	328
12.3.3	Spherical Container	333
13	Simulation of Resistivity Logging Devices	337
13.1	Description and Finite Element Modeling of Resistivity Logging Measurements	338
13.1.1	Laterolog Instruments	338
13.1.2	Through-Casing Instruments	340
13.1.3	Induction Instruments	341
13.1.4	Logging-While-Drilling (LWD) Instruments	343
13.1.5	Cross-Well Configurations and Surface-to-Borehole Measurements	344
13.1.6	Numerical Modeling of Antennas and Boundary Conditions	344
13.2	2D Numerical Simulations of Axisymmetric Problems	345
13.2.1	Model Formation	346
13.2.2	Model Logging Instruments	346
13.2.3	Numerical Simulations	349
13.2.4	Physical Interpretation	353
13.3	3D Numerical Simulations	361
14	Conclusions and Future Work	371
Appendix A		375
Appendix B		383
References		393
Index		403