

Contents

Notation	xi
Preface	xvii
1 Introduction	1
1.1 Definition of an Ideal Turbulence Model	2
1.2 How Complex Must a Turbulence Model Be?	2
1.3 Comments on the Physics of Turbulence	3
1.3.1 Importance of Turbulence in Practical Situations	3
1.3.2 General Properties of Turbulence	5
1.3.3 The Smallest Scales of Turbulence	10
1.3.4 Spectral Representation and the Kolmogorov $-5/3$ Law	11
1.3.5 The Law of the Wall	15
1.3.6 Power Laws	20
1.4 A Brief History of Turbulence Modeling	23
Problems	28
2 The Closure Problem	33
2.1 Reynolds Averaging	34
2.2 Correlations	39
2.3 Reynolds-Averaged Equations	39
2.4 The Reynolds-Stress Equation	41
2.5 The Scales of Turbulence	43
2.5.1 Turbulence Intensity	44
2.5.2 Two-Point Correlation Tensors and Related Scales	45
Problems	50
3 Algebraic Models	53
3.1 Molecular Transport of Momentum	54
3.2 The Mixing-Length Hypothesis	57
3.3 Application to Free Shear Flows	60

3.3.1	The Far Wake	62
3.3.2	The Mixing Layer	67
3.3.3	The Jet	70
3.4	Modern Variants of the Mixing-Length Model	74
3.4.1	Cebeci-Smith Model	79
3.4.2	Baldwin-Lomax Model	81
3.5	Application to Wall-Bounded Flows	84
3.5.1	Channel and Pipe Flow	84
3.5.2	Boundary Layers	89
3.6	Separated Flows	94
3.7	The 1/2-Equation Model	96
3.8	Range of Applicability	100
	Problems	102
4	One-Equation and Two-Equation Models	107
4.1	The Turbulence Energy Equation	108
4.2	One-Equation Models	111
4.3	Two-Equation Models	122
4.3.1	The k - ω Model	124
4.3.2	The k - ϵ Model	128
4.3.3	Other Two-Equation Models	131
4.4	Closure Coefficients	133
4.5	Application to Free Shear Flows	136
4.5.1	Developing the Similarity Solution	137
4.5.2	Numerical Solution	143
4.5.3	Sensitivity to Finite Freestream Boundary Conditions	147
4.5.4	Cross Diffusion	151
4.5.5	The Round-Jet/Plane-Jet Anomaly	154
4.6	Perturbation Analysis of the Boundary Layer	156
4.6.1	The Log Layer	156
4.6.2	The Defect Layer	161
4.6.3	The Viscous Sublayer	175
4.7	Surface Boundary Conditions	180
4.7.1	Wall Functions	181
4.7.2	Surface Roughness	182
4.7.3	Surface Mass Injection	186
4.8	Application to Wall-Bounded Flows	187
4.8.1	Channel and Pipe Flow	187
4.8.2	Boundary Layers	189
4.9	Low-Reynolds-Number Effects	192
4.9.1	Asymptotic Consistency	193
4.9.2	Transition	200

4.9.3	Channel and Pipe Flow	210
4.9.4	Boundary-Layer Applications	212
4.10	Application to Separated Flows	218
4.11	Range of Applicability	227
	Problems	230
5	Effects of Compressibility	239
5.1	Physical Considerations	239
5.2	Favre Averaging	241
5.3	Favre-Averaged Equations	243
5.4	Compressible-Flow Closure Approximations	249
5.4.1	Reynolds-Stress Tensor	250
5.4.2	Turbulent Heat-Flux Vector	250
5.4.3	Molecular Diffusion and Turbulent Transport	250
5.4.4	Dilatation Dissipation	251
5.4.5	Pressure Diffusion and Pressure Dilatation	253
5.4.6	Pressure Work	254
5.4.7	k - ω Model Equations for Compressible Flows	255
5.5	Mixing-Layer Compressibility Corrections	257
5.5.1	The Sarkar/Zeman/Wilcox Compressibility Corrections	258
5.5.2	Applications	259
5.6	Compressible Law of the Wall	262
5.6.1	Derivation	262
5.6.2	The Effect of Cross Diffusion	268
5.7	Compressible Boundary Layers	269
5.8	Shock-Induced Boundary-Layer Separation	275
5.8.1	The Earliest Applications	275
5.8.2	The Use of Wall Functions for Shock-Separated Flows	277
5.8.3	The Next Two Decades of “Progress”	279
5.8.4	Effect of the Stress Limiter on Shock-Separated Flows	280
5.8.5	Transonic Flow Over an Axisymmetric Bump	283
5.8.6	Mach 2 Flow Past a Backward-Facing Step	284
5.8.7	Mach 3 Compression Corners and Reflecting Shocks	285
5.8.8	Mach 11 Reflecting-Shock	289
5.8.9	The Reattachment Point Heat-Transfer Anomaly	290
5.8.10	Three-Dimensional Applications	292
5.9	Summary	295
	Problems	297

6	Beyond the Boussinesq Approximation	303
6.1	Boussinesq-Approximation Deficiencies	303
6.2	Nonlinear Constitutive Relations	308
6.2.1	The Earliest Formulations	308
6.2.2	Algebraic Stress Models	311
6.2.3	Relation to the Stress Limiter	317
6.2.4	Lag Model	320
6.3	Stress-Transport Models	322
6.3.1	Closure Approximations	323
6.3.2	Launder-Reece-Rodi Model	330
6.3.3	Wilcox Stress- ω Model	332
6.4	Application to Homogeneous Turbulent Flows	334
6.5	Application to Free Shear Flows	340
6.6	Application to Wall-Bounded Flows	343
6.6.1	Surface Boundary Conditions/Viscous Modifications	343
6.6.2	Channel and Pipe Flow	348
6.6.3	Rotating Channel Flow	351
6.6.4	Boundary Layers	352
6.7	Application to Separated Flows	361
6.7.1	Incompressible Backward-Facing Step	361
6.7.2	Transonic Flow Over an Axisymmetric Bump	365
6.7.3	Mach 3 Compression Corners and Reflecting Shocks	366
6.7.4	Hypersonic Shock-Separated Flows	370
6.8	Range of Applicability	371
	Problems	373
7	Numerical Considerations	381
7.1	Multiple Time Scales and Stiffness	381
7.2	Numerical Accuracy Near Boundaries	383
7.2.1	Solid Surfaces	383
7.2.2	Turbulent/Nonturbulent Interfaces	387
7.2.3	Sensitivity to Freestream Boundary Conditions	395
7.2.4	Viscous-Interface Layer	397
7.3	Parabolic Marching Methods	399
7.4	Elementary Time-Marching Methods	403
7.5	Block-Implicit Methods	409
7.6	Solution Convergence and Grid Sensitivity	414
7.6.1	Iteration Convergence and Grid Convergence	414
7.6.2	Richardson Extrapolation	416
7.6.3	Grid Convergence Index	417
7.6.4	Near-Wall Grid-Point Spacing	418
	Problems	420

8	New Horizons	427
8.1	Background Information	428
8.2	Direct Numerical Simulation	431
8.3	Large Eddy Simulation	436
8.3.1	Filtering	437
8.3.2	Subgrid-Scale (SGS) Modeling	440
8.3.3	“Off the Wall” Boundary Conditions	442
8.3.4	Applications	444
8.4	Detached Eddy Simulation	446
8.4.1	DES-Blending Functions	446
8.4.2	Applications	451
8.5	Chaos	452
8.6	Further Reading	455
	Problems	456
A	Cartesian Tensor Analysis	459
	Problems	464
B	Rudiments of Perturbation Methods	465
	Problems	475
C	Companion Software	477
	Bibliography	479
	Index	509