

CONTENTS

Notation	xiii
Introduction	1
I LOWER SEMICONTINUITY OF INTEGRAL FUNCTIONALS	
1 Lower semicontinuity and coerciveness	11
1.1 Lower semicontinuity	11
1.2 Yosida transforms	13
1.3 Coerciveness conditions. The direct method	16
1.4 Exercises	17
2 Weak convergence	18
2.1 Weak convergence in Lebesgue spaces	18
2.2 Weak convergence in Sobolev spaces	22
2.3 Weak* convergence of measures	22
2.4 Weak compactness criteria in L^1	24
2.5 Exercises	27
3 Minimum problems in Sobolev spaces	28
3.1 The direct method. An example of application	28
3.2 Borel and Carathéodory functions	29
3.3 Rellich's Theorem and equivalent conditions for lower semicontinuity	31
3.4 Exercises	32
4 Necessary conditions for weak lower semicontinuity	33
4.1 General necessary conditions	33
4.2 $W^{1,p}$ -quasiconvexity	34
4.3 Rank-1-convexity	40
4.4 Exercises	41
5 Sufficient conditions for weak lower semicontinuity	42
5.1 Convexity	42
5.2 Polyconvexity	45
5.3 Quasiconvexity	48
5.4 Exercises	52
6 The structure of quasiconvex functions	54
6.1 Quasiconvexity of polyconvex functions	54
6.2 Quasiconvexification	55

6.3	Example of a quasiconvex non-polyconvex function	59
6.4	Example of a rank-1-convex non-quasiconvex function	60
II Γ -CONVERGENCE		
7	A naïve introduction to Γ-convergence	65
7.1	Definition and basic properties	65
7.2	Lower and upper Γ -limits	67
7.3	Further properties. Compactness	70
7.4	Exercises	72
8	The indirect methods of Γ-convergence	73
8.1	Γ -limits and Yosida transforms	73
8.2	An example: Γ -limits of quadratic functionals	74
9	Direct methods. An integral representation result	77
9.1	Localization	77
9.2	Integral representation on Sobolev spaces	77
9.3	Integral representation of homogeneous functionals	81
10	Increasing set functions	82
10.1	Increasing set functions	82
10.2	A characterization of measures as set functions	82
10.3	Increasing set functions and compactness of Γ -limits	84
11	The fundamental estimate	85
11.1	Fundamental estimates	85
11.2	Subadditivity of Γ -limits	88
11.3	Γ -limits and boundary values	90
11.4	Exercises	92
12	Integral functionals with standard growth conditions	93
12.1	Standard growth conditions	93
12.2	Fundamental estimate	93
12.3	Compactness for the Γ -limits	95
12.4	Γ -limits of homogeneous functionals	96
12.5	Exercises	98
III BASIC HOMOGENIZATION		
13	A 1-dimensional example	101
13.1	The cell-problem homogenization formula	101
13.2	The asymptotic homogenization formula	103
13.3	Proof of the Γ -convergence	104
13.4	Exercises	106
14	Periodic homogenization	108
14.1	The asymptotic homogenization formula	109

14.2	The Homogenization Theorem	111
14.3	Convex homogenization	114
14.3.1	The cell-problem formula	114
14.3.2	Non-coercive convex homogenization	115
14.4	A counterexample to the cell-problem formula	120
14.5	An application: homogenization of elliptic equations in divergence form	123
14.6	Exercises	125
15	Almost-periodic homogenization	128
15.1	Homogenization of uniformly almost-periodic functionals	128
15.2	An example: loss of smoothness by homogenization	135
15.3	Exercises	140
16	Two applications	142
16.1	Homogenization of Riemannian metrics	142
16.2	Homogenization of Hamilton Jacobi equations	145
17	A closure theorem for the homogenization	150
17.1	A closure theorem	150
17.2	An application: homogenization of Besicovitch almost-periodic functionals	156
18	Loss of polyconvexity by homogenization	160
18.1	An example	160
IV FINER HOMOGENIZATION RESULTS		
19	Homogenization of connected media	167
19.1	A homogenization theorem on periodic connected domains	167
19.2	Convergence of Neumann boundary value problems	177
19.3	Convergence of Dirichlet boundary value problems	179
20	Homogenization with stiff and soft inclusions	181
20.1	Media with stiff and soft inclusions	181
20.2	The Homogenization Theorem	183
20.3	Convergence of minima	190
20.4	A Lavrentiev phenomenon	193
20.5	Loss of polyconvexity after homogenization	196
21	Homogenization with non-standard growth conditions	199
21.1	A class of non-standard integrals	199
21.2	Convex homogenization	202
21.3	Non-convex homogenization	203
21.4	Exercises	212

22 Iterated homogenization	214
22.1 Statement of the Iterated Homogenization Theorem	214
22.2 Proof of the Iterated Homogenization Theorem	215
22.3 Exercises	222
23 Correctors for the homogenization	227
23.1 Convergence of momenta in homogenization	227
23.2 Definition and some properties of the correctors	234
23.3 Statement and proof of the correctors result	240
23.4 Correctors in the quasiperiodic case	246
23.5 Exercises	248
24 Homogenization of multi-dimensional structures	249
24.1 A smooth approach	249
24.2 A measure Sobolev-space approach	253
24.3 Homogenization of periodic thin structures	263
24.4 Exercises	268
V APPENDICES	
A Almost-periodic functions	273
B Construction of extension operators	277
C Some regularity results	287
References	289
Notes to references	294
Index	297