

Contents

Preface	xi
1 Introduction	1
1.1 Goals and origins of $\mathcal{H}_\infty$ optimal control	1
1.2 Optimizing the command response	6
1.3 Optimal disturbance attenuation	9
1.3.1 Internal stability theory for stable plants	10
1.3.2 Solution of the disturbance attenuation problem	11
1.4 A robust stability problem	13
1.5 Concluding comments and references	17
1.6 Problems	19
2 Multivariable Frequency Response Design	21
2.1 Introduction	21
2.2 Singular values	28
2.2.1 The singular value decomposition	28
2.2.2 Singular value inequalities	31
2.3 Singular values and the sensitivity operator	34
2.4 Robust stability analysis	36
2.4.1 A Nyquist stability theorem	38
2.4.2 Additive model error	40
2.4.3 Multiplicative model error	44
2.4.4 Examples	47
2.5 Performance analysis and enhancement	52
2.5.1 Disturbance attenuation	53
2.5.2 Tracking	55
2.5.3 Sensor errors	55
2.5.4 The control signal	56
2.5.5 Robust performance	57
2.5.6 Analytic limits on performance	59

- 2.6 Example 62
- 2.7 Notes and References 65
- 2.8 Problems 68

3 Signals and Systems

71

- 3.1 Signals 72
 - 3.1.1 The size of signals 72
 - 3.1.2 Signals in the frequency domain 77
- 3.2 Systems 79
 - 3.2.1 Linear systems 80
 - 3.2.2 The space $\mathcal{L}_\infty$ 82
 - 3.2.3 The space $\mathcal{H}_\infty$ 84
 - 3.2.4 Adjoint systems 85
 - 3.2.5 Allpass systems 86
- 3.3 The size of a system 89
 - 3.3.1 The incremental gain 90
 - 3.3.2 The induced norm 91
 - 3.3.3 The 2-norm of a system 93
- 3.4 The small gain theorem 96
- 3.5 Loop transformation 98
 - 3.5.1 Multipliers or weights 99
 - 3.5.2 Linear shift 100
 - 3.5.3 Passivity 102
- 3.6 Robust stability revisited 106
- 3.7 The bounded real lemma 108
 - 3.7.1 An algebraic proof 111
 - 3.7.2 An optimal control proof 114
- 3.8 Notes and References 122
- 3.9 Problems 124

4 Linear Fractional Transformations

131

- 4.1 Introduction 131
 - 4.1.1 The composition formula 134
 - 4.1.2 Interconnections of state-space LFTs 135
- 4.2 LFTs in controller synthesis 138
 - 4.2.1 The generalized regulator problem 141
 - 4.2.2 The full-information problem 145
- 4.3 Contractive LFTs 150
 - 4.3.1 Constant matrix case 151
 - 4.3.2 Dynamic matrix case 153
- 4.4 Minimizing the norm of constant LFTs 157
- 4.5 Simplifying constant LFTs 160
- 4.6 Simplifying the generalized plant 164
- 4.7 Notes and References 170

4.8	Problems	172	
5	LQG Control		179
5.1	Introduction	179	
5.2	Full information	181	
5.2.1	The finite-horizon case	182	
5.2.2	The infinite-horizon case	188	
5.2.3	Inclusion of cross terms	194	
5.3	The Kalman filter	197	
5.3.1	The finite-horizon case	197	
5.3.2	The infinite-horizon case	201	
5.4	Measurement feedback	203	
5.4.1	The finite-horizon case	203	
5.4.2	The infinite-horizon case	205	
5.5	Notes and References	207	
5.6	Problems	208	
6	Full-Information $\mathcal{H}_\infty$ Controller Synthesis		215
6.1	Introduction	215	
6.2	The finite-horizon case	217	
6.2.1	Connection to differential games	218	
6.2.2	First-order necessary conditions	218	
6.2.3	The Riccati equation	221	
6.2.4	Sufficiency: completing the square	223	
6.2.5	Necessity	224	
6.2.6	All closed-loop systems	230	
6.2.7	All controllers	232	
6.3	The infinite-horizon case	235	
6.3.1	Preliminary observations	235	
6.3.2	Sufficiency	238	
6.3.3	A monotonicity property	238	
6.3.4	Assumptions	239	
6.3.5	Necessity	242	
6.3.6	All controllers	250	
6.4	Notes and References	251	
6.5	Problems	253	
7	The $\mathcal{H}_\infty$ Filter		263
7.1	Introduction	263	
7.2	Finite-horizon results	265	
7.2.1	Necessary and sufficient conditions	265	
7.2.2	All solutions	266	
7.2.3	Terminal state estimation properties	268	
7.3	Infinite-horizon results	270	

7.3.1	The $\mathcal{H}_\infty$ Wiener filtering problem	272
7.4	Example: Inertial navigation system	274
7.5	Notes and References	279
7.6	Problems	280
8	The $\mathcal{H}_\infty$ Generalized Regulator Problem	285
8.1	Introduction	285
8.1.1	Problem statement	286
8.2	Finite-horizon results	288
8.2.1	Two necessary conditions	289
8.2.2	Necessary and sufficient conditions	291
8.3	Infinite-horizon results	295
8.3.1	A necessary condition	295
8.3.2	An equivalent problem	296
8.3.3	Necessary and sufficient conditions	297
8.4	Example	299
8.5	Notes and References	302
8.6	Problems	304
9	Model Reduction by Truncation	313
9.1	Introduction	313
9.2	State-space truncation	315
9.2.1	The truncation error	317
9.2.2	Singular perturbation approximation	319
9.3	Balanced realization	321
9.3.1	Model reduction motivation	322
9.3.2	Balanced realization	323
9.4	Balanced truncation	325
9.4.1	Stability	326
9.4.2	Error bound for “one-step” truncation	328
9.4.3	The error bound for balanced truncation	330
9.5	Balanced singular perturbation approximation	332
9.6	Example	333
9.7	Notes and References	335
9.8	Problems	336
10	Optimal Model Reduction	341
10.1	Introduction	341
10.2	Hankel operators	341
10.2.1	The Hankel norm	342
10.2.2	Hankel singular values and the Schmidt decomposition	343
10.2.3	A lower bound on the approximation error	345
10.3	Suboptimal Hankel norm approximations	347
10.3.1	Allpass embedding	347

- 10.3.2 One solution to the model reduction problem 350
- 10.3.3 All solutions to the model reduction problem 351
- 10.4 Optimal Hankel norm approximation 352
 - 10.4.1 Optimal allpass embedding 352
 - 10.4.2 One optimal Hankel norm approximant 354
 - 10.4.3 All optimal approximants 355
 - 10.4.4 Nehari's theorem 359
- 10.5 The infinity norm error bound 360
 - 10.5.1 Hankel singular values of optimal error systems 360
 - 10.5.2 The error bound 362
- 10.6 Example 364
- 10.7 Notes and References 365
- 10.8 Problems 368

11 The Four-Block Problem

373

- 11.1 Introduction 373
- 11.2 The constant matrix problem 375
- 11.3 Suboptimal solutions 378
 - 11.3.1 The necessary conditions 378
 - 11.3.2 State-space construction of the dilation 382
 - 11.3.3 The sufficient conditions 385
- 11.4 Frequency weighted model reduction 389
 - 11.4.1 Problem formulation and a lower bound 391
 - 11.4.2 Reformulation as a four-block problem 392
 - 11.4.3 Allpass dilation 393
 - 11.4.4 Infinity norm error bounds 396
 - 11.4.5 Relative error model reduction 397
 - 11.4.6 Example 399
- 11.5 All $\mathcal{H}_\infty$ optimal controllers 401
- 11.6 Notes and References 409
- 11.7 Problems 410

12 Design Case Studies

415

- 12.1 Introduction 415
- 12.2 Robust stability 417
 - 12.2.1 Normalized coprime factor perturbations 418
 - 12.2.2 Loop-shaping design procedure 422
- 12.3 Tokamak plasma control 424
 - 12.3.1 The system model 425
 - 12.3.2 The design problem 427
 - 12.3.3 Control system design 429
 - 12.3.4 Antiwindup scheme 430
 - 12.3.5 Bumpless transfer scheme 433
 - 12.3.6 Simulations 434

- 12.4 High-purity distillation 437
 - 12.4.1 Design specification 439
 - 12.4.2 The system model 439
 - 12.4.3 Two-degree-of-freedom controller design 441
 - 12.4.4 Design weight selection 444
 - 12.4.5 Simulation results 446
- 12.5 Notes and References 448

Appendices

A Internal Stability Theory	453
A.1 Introduction 453	
A.1.1 Basics 455	
A.2 Coprime factorization 456	
A.2.1 Coprime factorization and internal stability 457	
A.2.2 Doubly coprime factorization 458	
A.3 All stabilizing controllers 460	
A.4 Internal stability of LFTs 464	
A.4.1 The full-information configuration 469	
A.5 Notes and References 470	
A.6 Problems 472	
 B Discrete-Time $\mathcal{H}_\infty$ Synthesis Theory	 475
B.1 Introduction 475	
B.1.1 Definitions 476	
B.1.2 Problem statement 478	
B.2 Full information 479	
B.2.1 Finite horizon 479	
B.2.2 Infinite horizon 487	
B.2.3 Convergence of the Riccati equation 489	
B.3 Filtering 494	
B.3.1 Finite horizon 495	
B.3.2 Infinite horizon 497	
B.4 Measurement feedback 497	
B.4.1 Finite horizon 498	
B.4.2 Infinite horizon 500	
B.5 Notes and References 501	
B.6 Problems 501	
 Bibliography	 507
 Index	 525