
Contents

Preface	xvii
1 Introduction	1
1.1 Mathematical Models	1
1.2 Numerical Simulations	3
1.3 The Finite Element Method	5
1.4 Nonlinear Analysis	7
1.4.1 Introduction	7
1.4.2 Classification of Nonlinearities	7
1.5 The Big Picture	11
References	12
2 The Finite Element Method: A Review	13
2.1 Introduction	13
2.2 One-Dimensional Problems	13
2.2.1 Governing Differential Equation	13
2.2.2 Finite Element Approximation	14
2.2.3 Derivation of the Weak Form	16
2.2.4 Interpolation Functions	18
2.2.5 Finite Element Model	22
2.3 Two-Dimensional Problems	24
2.3.1 Governing Differential Equation	24
2.3.2 Finite Element Approximation	24
2.3.3 Weak Formulation	26
2.3.4 Finite Element Model	27
2.3.5 Interpolation Functions	28
2.3.6 Assembly of Elements	33
2.4 Library of Two-Dimensional Finite Elements	36
2.4.1 Introduction	36
2.4.2 Triangular Elements	37
2.4.3 Rectangular Elements	38
2.5 Numerical Integration	40
2.5.1 Preliminary Comments	40
2.5.2 Coordinate Transformations	41
2.5.3 Integration Over a Master Rectangular Element	44
2.5.4 Integration Over a Master Triangular Element	45

2.6 Computer Implementation	46
2.6.1 General Comments	46
2.6.2 One-Dimensional Problems	48
2.6.3 Two-Dimensional Problems	52
2.7 Closure	53
Problems	57
References	59
3 Heat Transfer and Other Field Problems in One Dimension	61
3.1 Model Differential Equation	61
3.2 Weak Formulation	62
3.3 Finite Element Model	62
3.4 Solution Procedures	64
3.4.1 General Comments	64
3.4.2 Direct Iteration Procedure	65
3.4.3 Newton's Iteration Procedure	68
3.5 Computer Implementation	73
3.5.1 Introduction	73
3.5.2 Preprocessor Unit	73
3.5.3 Processor Unit	76
3.6 Closing Remarks	82
Problems	82
References	85
4 Nonlinear Bending of Straight Beams	87
4.1 Introduction	87
4.2 Euler–Bernoulli Beams	88
4.2.1 Basic Assumptions	88
4.2.2 Displacement Field and Strains	88
4.2.3 Weak Forms	89
4.2.4 Finite Element Model	95
4.2.5 Iterative Solutions of Nonlinear Equations	97
4.2.6 Load Increments	100
4.2.7 Membrane Locking	101
4.2.8 Computer Implementation	102
4.3 Timoshenko Beams	110
4.3.1 Displacement Field and Strains	110
4.3.2 Weak Forms	111
4.3.3 General Finite Element Model	113
4.3.4 Shear and Membrane Locking	115
4.3.5 Tangent Stiffness Matrix	117

Problems	124
References	126
5 Heat Transfer and Other Fields Problems in Two Dimensions	127
5.1 Model Equation	127
5.2 Weak Form	128
5.3 Finite Element Model	129
5.4 Solution Procedures	131
5.4.1 Direct Iteration	131
5.4.2 Newton–Raphson Iteration	131
5.5 Computer Implementation	132
5.5.1 Introduction	132
5.5.2 Numerical Integration	133
5.5.3 Element Calculations	135
Problems	139
References	140
6 Nonlinear Bending of Elastic Plates	141
6.1 Introduction	141
6.2 Classical Plate Theory	141
6.2.1 Assumptions of the Kinematics	141
6.2.2 Displacement Field and Strains	142
6.3 Variational Formulation of CPT	144
6.3.1 Virtual Work	144
6.3.2 Weak Forms	147
6.3.3 Equilibrium Equations	148
6.3.4 Boundary Conditions	149
6.3.5 Stress Resultant–Deflection Relations	152
6.4 Finite Element Models of CPT	153
6.4.1 General Formulation	153
6.4.2 Tangent Stiffness Coefficients	157
6.4.3 Some Plate Finite Elements	161
6.5 Computer Implementation Aspects and Numerical Results of CPT Elements	164
6.5.1 Computer Implementation	164
6.5.2 Results of Linear Analysis	166
6.5.3 Results of Nonlinear Analysis	168
6.6 First–Order Shear Deformation Plate Theory	173
6.6.1 Introduction	173
6.6.2 Displacement Field	173
6.6.3 Weak Formulation	174

6.7 Finite Element Models of FSDT	177
6.7.1 Virtual Work Statements	177
6.7.2 Finite Element Model	178
6.7.3 Tangent Stiffness Coefficients	182
6.7.4 Shear and Membrane Locking	184
6.8 Computer Implementation and Numerical Results of FSDT Elements	184
6.8.1 Computer Implementation	184
6.8.2 Results of Linear Analysis	185
6.8.3 Results of Nonlinear Analysis	189
6.9 Theory of Doubly-Curved Shells	196
6.9.1 Introduction	196
6.9.2 Geometric Description	197
6.9.3 Strain–Displacement Relations	201
6.9.4 Stress Resultants	202
6.9.5 Equations of Motion	205
6.10 Finite Element Analysis of Shells	206
6.10.1 Weak Forms	206
6.10.2 Finite Element Model	207
6.10.3 Linear Results	209
6.10.4 Nonlinear Results	217
Problems	222
References	225
7 Flows of Viscous Incompressible Fluids	229
7.1 Introduction	229
7.2 Governing Equations	230
7.2.1 Introduction	230
7.2.2 Conservation of Mass	231
7.2.3 Conservation of Momenta	232
7.2.4 Conservation of Energy	232
7.2.5 Constitutive Equations	233
7.2.6 Boundary Conditions	234
7.3 Governing Equations in Terms of Primitive Variables	235
7.3.1 Vector Form	235
7.3.2 Cartesian Component Form	236
7.4 Velocity–Pressure Finite Element Model	237
7.4.1 Weak Form	237
7.4.2 Finite Element Model	239
7.5 Penalty Finite Element Models	241
7.5.1 Introduction	241
7.5.2 Penalty Function Method	242

7.5.3 Reduced Integration Penalty Model	244
7.5.4 Consistent Penalty Model	245
7.6 Computational Aspects	246
7.6.1 Properties of the Matrix Equations	246
7.6.2 Choice of Elements	247
7.6.3 Evaluation of Element Matrices in Penalty Models	249
7.6.4 Post-Computation of Stresses	250
7.7 Computer Implementation	251
7.7.1 Mixed Model	251
7.7.2 Penalty Model	254
7.8 Numerical Examples	254
7.8.1 Preliminary Comments	254
7.8.2 Fluid Squeezed Between Parallel Plates	255
7.8.3 Flow of a Viscous Lubricant in a Slider Bearing	258
7.8.4 Wall-Driven Cavity Flow	261
7.8.5 Backward-Facing Step	267
7.9 Least-Squares Finite Element Models	269
7.9.1 Introduction	269
7.9.2 Finite Element Model	272
7.9.3 Computational Aspects	274
7.9.4 Numerical Examples	275
Problems	283
References	283
8 Nonlinear Analysis of Time-Dependent Problems	287
8.1 Introduction	287
8.2 Time Approximations	288
8.2.1 Introduction	288
8.2.2 Parabolic Equations	289
8.2.3 Hyperbolic Equations	292
8.3 Stability and Accuracy	295
8.3.1 Preliminary Comments	295
8.3.2 Stability Criteria	296
8.4 Transient Analysis of Nonlinear Problems	297
8.4.1 Introduction	297
8.4.2 Heat Transfer	297
8.4.3 Flows of Viscous Incompressible Fluids	298
8.4.4 Plate Bending (FSDT)	300
8.5 Computer Implementation	304

8.6 Numerical Examples	307
8.6.1 Linear Problems	307
8.6.2 Nonlinear Problems	314
Problems	323
References	325
9 Finite Element Formulations of Solid Continua	327
9.1 Introduction	327
9.1.1 Background	327
9.1.2 Descriptions of Motion	328
9.2 Strain and Stress Measures	329
9.2.1 Deformation Gradient Tensor	329
9.2.2 Green and Almansi Strain Tensors	331
9.2.3 Polar Decomposition	334
9.2.4 Stress Tensors	334
9.2.5 Energetically-Conjugate Stresses and Strains	335
9.3 Strain and Stress Measures Between Configurations	337
9.3.1 Notation	337
9.3.2 Conservation of Mass	339
9.3.3 Green Strain Tensors for Various Configurations	340
9.3.4 Euler Strain Tensor	342
9.3.5 Relationships Between Various Stress Tensors	343
9.4 Constitutive Equations	346
9.5 Total Lagrangian and Updated Lagrangian Formulations of Continua	347
9.5.1 Principle of Virtual Displacements	347
9.5.2 Total Lagrangian Formulation	348
9.5.3 Updated Lagrangian Formulation	350
9.6 Finite Element Models of Two-Dimensional Continua	353
9.6.1 Introduction	353
9.6.2 Total Lagrangian Formulation	353
9.6.3 Updated Lagrangian Formulation	360
9.6.4 Computer Implementation	361
9.6.5 Numerical Results	361
9.7 Shell Finite Elements	369
9.7.1 Introduction	369
9.7.2 Incremental Equations of Motion	369
9.7.3 Finite Element Models of a Continuum	370
9.7.4 Shell Finite Element	372
9.7.5 Numerical Examples	378
Problems	381
References	387

10 Material Nonlinearities and Coupled Problems	389
10.1 Introduction	389
10.2 Nonlinear Elastic Problems	390
10.3 Small Deformation Theory of Plasticity	391
10.3.1 Introduction	391
10.3.2 Ideal Plasticity	392
10.3.3 Strain Hardening Plasticity	393
10.3.4 Elastic–Plastic Analysis of a Bar	395
10.4 Non-Newtonian Fluids	404
10.4.1 Introduction	404
10.4.2 Governing Equations in Cylindrical Coordinates	405
10.4.3 Power-Law Fluids	407
10.4.4 White–Metzner Fluids	409
10.4.5 Numerical Examples	412
10.5 Coupled Fluid Flow and Heat Transfer	417
10.5.1 Finite Element Models	417
10.5.2 Numerical Examples	419
References	423
 Appendix A1: Solution Procedures for Linear Algebraic Equations	 425
A1.1 Introduction	425
A1.2 Direct Methods	427
A1.2.1 Preliminary Comments	427
A1.2.2 Symmetric Solver	428
A1.2.3 Unsymmetric Solver	430
A1.3 Iterative Methods	430
A1.3.1 General Comments	430
A1.3.2 Solution Algorithms	432
References	433
 Appendix A2: Solution Procedures for Nonlinear Algebraic Equations	 439
A2.1 Introduction	439
A2.2 Picard Iteration Method	440
A2.3 Newton–Raphson Iteration Method	444
A2.4 Riks and Modified Riks Schemes	448
References	457
 Subject Index	 459