

Contents

1	Introduction	1
2	Peridynamic Differential Operator	5
2.1	Concept of Peridynamics	6
2.2	PDDO for a Function with a Single Variable	7
2.3	PDDO for a Function $f(x)$ with Many Variables	10
2.4	PDDO for a Function $f(x)$ up to Second-Order Derivatives	12
2.4.1	PDDO for a Function $f(\mathbf{x})$ with Three Variables	13
2.4.2	PDDO for a Function $f(\mathbf{x})$ with Two Variables	24
2.5	PDDO for a Relative Function $f(\mathbf{x}+\boldsymbol{\xi}) - f(\mathbf{x})$ up to Second-Order Derivatives	31
2.5.1	Nonsymmetric Family with Three Variables	31
2.5.2	Symmetric Family with Three Variables	33
2.5.3	Nonsymmetric Family with Two Variables	35
2.5.4	Symmetric Family with Two Variables	36
3	Numerical Implementation	39
3.1	Discretization	39
3.2	Discrete Form of Differential Equations	42
3.3	Sources of Numerical Error	44
4	Interpolation, Regression, and Smoothing	57
4.1	PDDO for Estimating a Function	58
4.2	PD Interpolation	59
4.2.1	One-Dimensional Interpolation	59
4.2.2	Two-Dimensional Interpolation	65
4.3	Image Recovery	71
4.4	Adaptive Image Compression	72
4.5	PD Regression	75
4.5.1	One-Dimensional Regression	77
4.5.2	Two-Dimensional Regression	78

4.6	PD Smoothing	80
4.6.1	Noise Removal	81
4.6.2	Recursive Smoothing	83
4.7	PD Image Enhancement	86
5	Ordinary Differential Equations	91
5.1	Initial Value Problems	91
5.1.1	Nonlinear Pendulum Equation	91
5.1.2	Lane-Emden Equation	93
5.1.3	Stiff Coupled Equations	95
5.2	Boundary Value Problems	99
5.2.1	Ordinary Differential Equation with Variable Coefficients	99
5.2.2	Singular Perturbation	100
5.2.3	Boundary Layer Equation	103
5.2.4	Beam Equation with a Nonlinear Boundary Condition	105
5.2.5	Natural Convection on a Vertical Plate	107
5.2.6	Beam Buckling	110
5.2.7	Large Deflection of a Beam	113
6	Partial Differential Equations	117
6.1	Heat Diffusion in a Bar	118
6.2	Diffusion in a Plate	120
6.3	Diffusion in a Bar with Varying Diffusivity	123
6.4	Diffusion with Phase Change	126
6.5	Wave Propagation in a Bar	132
6.6	Helmholtz Equation	135
6.7	Steady-State Heat Conduction in a Plate with an Insulated Crack	138
6.8	Nonlinear Steady-State Heat Equation	141
6.9	Plate Bending Under Transverse Load	145
6.10	Plate with a Crack Under Tension	148
7	Coupled Field Equations	159
7.1	Nonlinear Burger's Equation	159
7.2	Thermoelastic Response of a Bar Under Dynamic Load	163
7.3	Thermoelectricity in a Bar	169
7.4	Thermo-oxidation in a Resin Strip	172
7.5	Lithium Diffusion and Stress Evolution in an Electrode with Cracks	175

8	Integrodifferential Equations	187
8.1	Abel's Integral Equation	188
8.1.1	Abel's IE with Continuous and Smooth Discrete Data	189
8.1.2	Abel's IE with Discontinuous and Smooth Discrete Data	190
8.2	Variational Iteration Method	193
8.2.1	Fredholm Integrodifferential Equation	195
8.2.2	Fredholm Integral Equation	197
8.2.3	Volterra Integral Equation	199
8.2.4	Third-Order Nonlinear ODE	201
8.2.5	Van der Pol Equation	204
9	Weak Form of Peridynamics	209
9.1	PD Computational Domain	209
9.2	Weak Form of PD Poisson's Equation	211
9.3	Heat Conduction in a Plate	223
9.4	PD Form of Navier's Equation	224
9.5	Weak Form of PD Navier's Equation	227
9.6	Plate with a Fixed Edge Under Applied Traction Loading	239
9.7	Plate with a Pre-existing Crack Under Tension	241
10	Peridynamic Least Square Minimization	247
10.1	PD-LSM Differentiation of a Function with Single Variable	249
10.2	PD-LSM Differentiation of a Function with Multivariables	250
10.3	Spherical Family and Symmetric Position	259
	Appendices	263
	References	275
	Index	279