
Contents

Preface	v
Notation	ix
Contents	xi
1 Introduction	1
1.1 Optimization with Multiple Criteria	1
1.2 Decision Space and Objective (Criterion) Space	4
1.3 Notions of Optimality	7
1.4 Orders and Cones	8
1.5 Classification of Multicriteria Optimization Problems	16
1.6 Notes	19
Exercises	20
2 Efficiency and Nondominance	23
2.1 Efficient Solutions and Nondominated Points	23
2.2 Bounds on the Nondominated Set	33
2.3 Weakly and Strictly Efficient Solutions	38
2.4 Proper Efficiency and Proper Nondominance	50
2.5 Notes	59
Exercises	62
3 The Weighted Sum Method and Related Topics	65
3.1 Weighted Sum Scalarization and (Weak) Efficiency	68
3.2 Weighted Sum Scalarization and Proper Efficiency	71
3.3 Optimality Conditions	80
3.4 Connectedness of Efficient and Nondominated Sets	86
3.5 Notes	91

Exercises	93
4 Scalarization Techniques	97
4.1 The ε -Constraint Method	98
4.2 The Hybrid Method	101
4.3 The Elastic Constraint Method	102
4.4 Benson's Method	106
4.5 Compromise Solutions – Approximation of the Ideal Point	111
4.6 The Achievement Function Method	121
4.7 Notes	122
Exercises	124
5 Other Definitions of Optimality – Nonscalarizing Methods	127
5.1 Lexicographic Optimality	128
5.2 Max-Ordering Optimality	132
5.3 Lexicographic Max-Ordering Optimization	135
5.4 Notes	146
Exercises	148
6 Introduction to Multicriteria Linear Programming	151
6.1 Notation and Definitions	152
6.2 The Simplex Method and Biobjective Linear Programs	160
Exercises	170
7 A Multiobjective Simplex Method	171
7.1 Algebra of Multiobjective Linear Programming	171
7.2 The Geometry of Multiobjective Linear Programming	184
7.3 Notes	193
Exercises	195
8 Multiobjective Combinatorial Optimization	197
8.1 Basics	198
8.2 Problems with Explicitly Given Feasible Set	209
8.3 Scalarization of Multiobjective Integer Programs	214
8.4 Notes	218
Exercises	220
9 Multiobjective Versions of Polynomially Solvable Problems	221
9.1 Algorithms for the Shortest Path Problem	221
9.2 The Minimum Spanning Tree Problem	236
9.3 Matroids	247
9.4 The Assignment Problem and the Two Phase Method	253

9.5	Notes	267
	Exercises	269
10	Multiobjective Versions of Some $\mathcal{NP}$-Hard Problems	271
10.1	The Knapsack Problem and Branch and Bound	271
10.2	The Travelling Salesperson Problem and Heuristics	279
10.3	Notes	288
	Exercises	290
	Bibliography	291
	List of Figures	307
	List of Tables	311
	Author Index	313
	Subject Index	315