

Contents

| | |
|------------------------|-------|
| <i>Preface</i> | xvii |
| <i>Acknowledgments</i> | xxiii |

I Introduction to Queueing

| | |
|--|-----------|
| 1 Motivating Examples of the Power of Analytical Modeling | 3 |
| 1.1 What Is Queueing Theory? | 3 |
| 1.2 Examples of the Power of Queueing Theory | 5 |
| 2 Queueing Theory Terminology | 13 |
| 2.1 Where We Are Heading | 13 |
| 2.2 The Single-Server Network | 13 |
| 2.3 Classification of Queueing Networks | 16 |
| 2.4 Open Networks | 16 |
| 2.5 More Metrics: Throughput and Utilization | 17 |
| 2.6 Closed Networks | 20 |
| 2.6.1 Interactive (Terminal-Driven) Systems | 21 |
| 2.6.2 Batch Systems | 22 |
| 2.6.3 Throughput in a Closed System | 23 |
| 2.7 Differences between Closed and Open Networks | 24 |
| 2.7.1 A Question on Modeling | 25 |
| 2.8 Related Readings | 25 |
| 2.9 Exercises | 26 |

II Necessary Probability Background

| | |
|---|-----------|
| 3 Probability Review | 31 |
| 3.1 Sample Space and Events | 31 |
| 3.2 Probability Defined on Events | 32 |
| 3.3 Conditional Probabilities on Events | 33 |
| 3.4 Independent Events and Conditionally Independent Events | 34 |
| 3.5 Law of Total Probability | 35 |
| 3.6 Bayes Law | 36 |
| 3.7 Discrete versus Continuous Random Variables | 37 |
| 3.8 Probabilities and Densities | 38 |
| 3.8.1 Discrete: Probability Mass Function | 38 |
| 3.8.2 Continuous: Probability Density Function | 41 |
| 3.9 Expectation and Variance | 44 |
| 3.10 Joint Probabilities and Independence | 47 |

| | | |
|---------------|---|-----------|
| 3.11 | Conditional Probabilities and Expectations | 49 |
| 3.12 | Probabilities and Expectations via Conditioning | 53 |
| 3.13 | Linearity of Expectation | 54 |
| 3.14 | Normal Distribution | 57 |
| 3.14.1 | Linear Transformation Property | 58 |
| 3.14.2 | Central Limit Theorem | 61 |
| 3.15 | Sum of a Random Number of Random Variables | 62 |
| 3.16 | Exercises | 64 |
| 4 | Generating Random Variables for Simulation | 70 |
| 4.1 | Inverse-Transform Method | 70 |
| 4.1.1 | The Continuous Case | 70 |
| 4.1.2 | The Discrete Case | 72 |
| 4.2 | Accept-Reject Method | 72 |
| 4.2.1 | Discrete Case | 73 |
| 4.2.2 | Continuous Case | 75 |
| 4.2.3 | Some Harder Problems | 77 |
| 4.3 | Readings | 78 |
| 4.4 | Exercises | 78 |
| 5 | Sample Paths, Convergence, and Averages | 79 |
| 5.1 | Convergence | 79 |
| 5.2 | Strong and Weak Laws of Large Numbers | 83 |
| 5.3 | Time Average versus Ensemble Average | 84 |
| 5.3.1 | Motivation | 85 |
| 5.3.2 | Definition | 86 |
| 5.3.3 | Interpretation | 86 |
| 5.3.4 | Equivalence | 88 |
| 5.3.5 | Simulation | 90 |
| 5.3.6 | Average Time in System | 90 |
| 5.4 | Related Readings | 91 |
| 5.5 | Exercise | 91 |

III The Predictive Power of Simple Operational Laws: “What-If” Questions and Answers

| | | |
|--------------|--|-----------|
| 6 | Little’s Law and Other Operational Laws | 95 |
| 6.1 | Little’s Law for Open Systems | 95 |
| 6.2 | Intuitions | 96 |
| 6.3 | Little’s Law for Closed Systems | 96 |
| 6.4 | Proof of Little’s Law for Open Systems | 97 |
| 6.4.1 | Statement via Time Averages | 97 |
| 6.4.2 | Proof | 98 |
| 6.4.3 | Corollaries | 100 |
| 6.5 | Proof of Little’s Law for Closed Systems | 101 |
| 6.5.1 | Statement via Time Averages | 101 |
| 6.5.2 | Proof | 102 |
| 6.6 | Generalized Little’s Law | 102 |

| | | |
|-------------|--|------------|
| 6.7 | Examples Applying Little's Law | 103 |
| 6.8 | More Operational Laws: The Forced Flow Law | 106 |
| 6.9 | Combining Operational Laws | 107 |
| 6.10 | Device Demands | 110 |
| 6.11 | Readings and Further Topics Related to Little's Law | 111 |
| 6.12 | Exercises | 111 |
| 7 | Modification Analysis: “What-If” for Closed Systems | 114 |
| 7.1 | Review | 114 |
| 7.2 | Asymptotic Bounds for Closed Systems | 115 |
| 7.3 | Modification Analysis for Closed Systems | 118 |
| 7.4 | More Modification Analysis Examples | 119 |
| 7.5 | Comparison of Closed and Open Networks | 122 |
| 7.6 | Readings | 122 |
| 7.7 | Exercises | 122 |

IV From Markov Chains to Simple Queues

| | | |
|--------------|--|------------|
| 8 | Discrete-Time Markov Chains | 129 |
| 8.1 | Discrete-Time versus Continuous-Time Markov Chains | 130 |
| 8.2 | Definition of a DTMC | 130 |
| 8.3 | Examples of Finite-State DTMCs | 131 |
| 8.3.1 | Repair Facility Problem | 131 |
| 8.3.2 | Umbrella Problem | 132 |
| 8.3.3 | Program Analysis Problem | 132 |
| 8.4 | Powers of \mathbf{P} : n -Step Transition Probabilities | 133 |
| 8.5 | Stationary Equations | 135 |
| 8.6 | The Stationary Distribution Equals the Limiting Distribution | 136 |
| 8.7 | Examples of Solving Stationary Equations | 138 |
| 8.7.1 | Repair Facility Problem with Cost | 138 |
| 8.7.2 | Umbrella Problem | 139 |
| 8.8 | Infinite-State DTMCs | 139 |
| 8.9 | Infinite-State Stationarity Result | 140 |
| 8.10 | Solving Stationary Equations in Infinite-State DTMCs | 142 |
| 8.11 | Exercises | 145 |
| 9 | Ergodicity Theory | 148 |
| 9.1 | Ergodicity Questions | 148 |
| 9.2 | Finite-State DTMCs | 149 |
| 9.2.1 | Existence of the Limiting Distribution | 149 |
| 9.2.2 | Mean Time between Visits to a State | 153 |
| 9.2.3 | Time Averages | 155 |
| 9.3 | Infinite-State Markov Chains | 155 |
| 9.3.1 | Recurrent versus Transient | 156 |
| 9.3.2 | Infinite Random Walk Example | 160 |
| 9.3.3 | Positive Recurrent versus Null Recurrent | 162 |
| 9.4 | Ergodic Theorem of Markov Chains | 164 |

| | | |
|---------------|---|------------|
| 9.5 | Time Averages | 166 |
| 9.6 | Limiting Probabilities Interpreted as Rates | 168 |
| 9.7 | Time-Reversibility Theorem | 170 |
| 9.8 | When Chains Are Periodic or Not Irreducible | 171 |
| 9.8.1 | Periodic Chains | 171 |
| 9.8.2 | Chains that Are Not Irreducible | 177 |
| 9.9 | Conclusion | 177 |
| 9.10 | Proof of Ergodic Theorem of Markov Chains* | 178 |
| 9.11 | Exercises | 183 |
| 10 | Real-World Examples: Google, Aloha, and Harder Chains* | 190 |
| 10.1 | Google's PageRank Algorithm | 190 |
| 10.1.1 | Google's DTMC Algorithm | 190 |
| 10.1.2 | Problems with Real Web Graphs | 192 |
| 10.1.3 | Google's Solution to Dead Ends and Spider Traps | 194 |
| 10.1.4 | Evaluation of the PageRank Algorithm | 195 |
| 10.1.5 | Practical Implementation Considerations | 195 |
| 10.2 | Aloha Protocol Analysis | 195 |
| 10.2.1 | The Slotted Aloha Protocol | 196 |
| 10.2.2 | The Aloha Markov Chain | 196 |
| 10.2.3 | Properties of the Aloha Markov Chain | 198 |
| 10.2.4 | Improving the Aloha Protocol | 199 |
| 10.3 | Generating Functions for Harder Markov Chains | 200 |
| 10.3.1 | The z-Transform | 201 |
| 10.3.2 | Solving the Chain | 201 |
| 10.4 | Readings and Summary | 203 |
| 10.5 | Exercises | 204 |
| 11 | Exponential Distribution and the Poisson Process | 206 |
| 11.1 | Definition of the Exponential Distribution | 206 |
| 11.2 | Memoryless Property of the Exponential | 207 |
| 11.3 | Relating Exponential to Geometric via δ -Steps | 209 |
| 11.4 | More Properties of the Exponential | 211 |
| 11.5 | The Celebrated Poisson Process | 213 |
| 11.6 | Merging Independent Poisson Processes | 218 |
| 11.7 | Poisson Splitting | 218 |
| 11.8 | Uniformity | 221 |
| 11.9 | Exercises | 222 |
| 12 | Transition to Continuous-Time Markov Chains | 225 |
| 12.1 | Defining CTMCs | 225 |
| 12.2 | Solving CTMCs | 229 |
| 12.3 | Generalization and Interpretation | 232 |
| 12.3.1 | Interpreting the Balance Equations for the CTMC | 234 |
| 12.3.2 | Summary Theorem for CTMCs | 234 |
| 12.4 | Exercises | 234 |

| | |
|---|------------|
| 13 M/M/1 and PASTA | 236 |
| 13.1 The M/M/1 Queue | 236 |
| 13.2 Examples Using an M/M/1 Queue | 239 |
| 13.3 PASTA | 242 |
| 13.4 Further Reading | 245 |
| 13.5 Exercises | 245 |
| V Server Farms and Networks: Multi-server, Multi-queue Systems | |
| 14 Server Farms: M/M/k and M/M/k/k | 253 |
| 14.1 Time-Reversibility for CTMCs | 253 |
| 14.2 M/M/k/k Loss System | 255 |
| 14.3 M/M/k | 258 |
| 14.4 Comparison of Three Server Organizations | 263 |
| 14.5 Readings | 264 |
| 14.6 Exercises | 264 |
| 15 Capacity Provisioning for Server Farms | 269 |
| 15.1 What Does Load Really Mean in an M/M/k? | 269 |
| 15.2 The M/M/ ∞ | 271 |
| 15.2.1 Analysis of the M/M/ ∞ | 271 |
| 15.2.2 A First Cut at a Capacity Provisioning Rule for the M/M/k | 272 |
| 15.3 Square-Root Staffing | 274 |
| 15.4 Readings | 276 |
| 15.5 Exercises | 276 |
| 16 Time-Reversibility and Burke's Theorem | 282 |
| 16.1 More Examples of Finite-State CTMCs | 282 |
| 16.1.1 Networks with Finite Buffer Space | 282 |
| 16.1.2 Batch System with M/M/2 I/O | 284 |
| 16.2 The Reverse Chain | 285 |
| 16.3 Burke's Theorem | 288 |
| 16.4 An Alternative (Partial) Proof of Burke's Theorem | 290 |
| 16.5 Application: Tandem Servers | 291 |
| 16.6 General Acyclic Networks with Probabilistic Routing | 293 |
| 16.7 Readings | 294 |
| 16.8 Exercises | 294 |
| 17 Networks of Queues and Jackson Product Form | 297 |
| 17.1 Jackson Network Definition | 297 |
| 17.2 The Arrival Process into Each Server | 298 |
| 17.3 Solving the Jackson Network | 300 |
| 17.4 The Local Balance Approach | 301 |
| 17.5 Readings | 306 |
| 17.6 Exercises | 306 |
| 18 Classed Network of Queues | 311 |
| 18.1 Overview | 311 |
| 18.2 Motivation for Classed Networks | 311 |

| | | |
|--|---|------------|
| 18.3 | Notation and Modeling for Classed Jackson Networks | 314 |
| 18.4 | A Single-Server Classed Network | 315 |
| 18.5 | Product Form Theorems | 317 |
| 18.6 | Examples Using Classed Networks | 322 |
| 18.6.1 | Connection-Oriented ATM Network Example | 322 |
| 18.6.2 | Distribution of Job Classes Example | 325 |
| 18.6.3 | CPU-Bound and I/O-Bound Jobs Example | 326 |
| 18.7 | Readings | 329 |
| 18.8 | Exercises | 329 |
| 19 | Closed Networks of Queues | 331 |
| 19.1 | Motivation | 331 |
| 19.2 | Product-Form Solution | 333 |
| 19.2.1 | Local Balance Equations for Closed Networks | 333 |
| 19.2.2 | Example of Deriving Limiting Probabilities | 335 |
| 19.3 | Mean Value Analysis (MVA) | 337 |
| 19.3.1 | The Arrival Theorem | 338 |
| 19.3.2 | Iterative Derivation of Mean Response Time | 340 |
| 19.3.3 | An MVA Example | 341 |
| 19.4 | Readings | 343 |
| 19.5 | Exercises | 343 |
| VI Real-World Workloads: High Variability and Heavy Tails | | |
| 20 | Tales of Tails: A Case Study of Real-World Workloads | 349 |
| 20.1 | Grad School Tales . . . Process Migration | 349 |
| 20.2 | UNIX Process Lifetime Measurements | 350 |
| 20.3 | Properties of the Pareto Distribution | 352 |
| 20.4 | The Bounded Pareto Distribution | 353 |
| 20.5 | Heavy Tails | 354 |
| 20.6 | The Benefits of Active Process Migration | 354 |
| 20.7 | Pareto Distributions Are Everywhere | 355 |
| 20.8 | Exercises | 357 |
| 21 | Phase-Type Distributions and Matrix-Analytic Methods | 359 |
| 21.1 | Representing General Distributions by Exponentials | 359 |
| 21.2 | Markov Chain Modeling of PH Workloads | 364 |
| 21.3 | The Matrix-Analytic Method | 366 |
| 21.4 | Analysis of Time-Varying Load | 367 |
| 21.4.1 | High-Level Ideas | 367 |
| 21.4.2 | The Generator Matrix, Q | 368 |
| 21.4.3 | Solving for R | 370 |
| 21.4.4 | Finding $\vec{\pi}_0$ | 371 |
| 21.4.5 | Performance Metrics | 372 |
| 21.5 | More Complex Chains | 372 |
| 21.6 | Readings and Further Remarks | 376 |
| 21.7 | Exercises | 376 |

| | |
|--|------------|
| 22 Networks with Time-Sharing (PS) Servers (BCMP) | 380 |
| 22.1 Review of Product-Form Networks | 380 |
| 22.2 BCMP Result | 380 |
| 22.2.1 Networks with FCFS Servers | 381 |
| 22.2.2 Networks with PS Servers | 382 |
| 22.3 M/M/1/PS | 384 |
| 22.4 M/Cox/1/PS | 385 |
| 22.5 Tandem Network of M/G/1/PS Servers | 391 |
| 22.6 Network of PS Servers with Probabilistic Routing | 393 |
| 22.7 Readings | 394 |
| 22.8 Exercises | 394 |
| 23 The M/G/1 Queue and the Inspection Paradox | 395 |
| 23.1 The Inspection Paradox | 395 |
| 23.2 The M/G/1 Queue and Its Analysis | 396 |
| 23.3 Renewal-Reward Theory | 399 |
| 23.4 Applying Renewal-Reward to Get Expected Excess | 400 |
| 23.5 Back to the Inspection Paradox | 402 |
| 23.6 Back to the M/G/1 Queue | 403 |
| 23.7 Exercises | 405 |
| 24 Task Assignment Policies for Server Farms | 408 |
| 24.1 Task Assignment for FCFS Server Farms | 410 |
| 24.2 Task Assignment for PS Server Farms | 419 |
| 24.3 Optimal Server Farm Design | 424 |
| 24.4 Readings and Further Follow-Up | 428 |
| 24.5 Exercises | 430 |
| 25 Transform Analysis | 433 |
| 25.1 Definitions of Transforms and Some Examples | 433 |
| 25.2 Getting Moments from Transforms: Peeling the Onion | 436 |
| 25.3 Linearity of Transforms | 439 |
| 25.4 Conditioning | 441 |
| 25.5 Distribution of Response Time in an M/M/1 | 443 |
| 25.6 Combining Laplace and z-Transforms | 444 |
| 25.7 More Results on Transforms | 445 |
| 25.8 Readings | 446 |
| 25.9 Exercises | 446 |
| 26 M/G/1 Transform Analysis | 450 |
| 26.1 The z-Transform of the Number in System | 450 |
| 26.2 The Laplace Transform of Time in System | 454 |
| 26.3 Readings | 456 |
| 26.4 Exercises | 456 |
| 27 Power Optimization Application | 457 |
| 27.1 The Power Optimization Problem | 457 |
| 27.2 Busy Period Analysis of M/G/1 | 459 |
| 27.3 M/G/1 with Setup Cost | 462 |

| | | |
|--|--|------------|
| 27.4 | Comparing ON/IDLE versus ON/OFF | 465 |
| 27.5 | Readings | 467 |
| 27.6 | Exercises | 467 |
| VII Smart Scheduling in the M/G/1 | | |
| 28 | Performance Metrics | 473 |
| 28.1 | Traditional Metrics | 473 |
| 28.2 | Commonly Used Metrics for Single Queues | 474 |
| 28.3 | Today's Trendy Metrics | 474 |
| 28.4 | Starvation/Fairness Metrics | 475 |
| 28.5 | Deriving Performance Metrics | 476 |
| 28.6 | Readings | 477 |
| 29 | Scheduling: Non-Preemptive, Non-Size-Based Policies | 478 |
| 29.1 | FCFS, LCFS, and RANDOM | 478 |
| 29.2 | Readings | 481 |
| 29.3 | Exercises | 481 |
| 30 | Scheduling: Preemptive, Non-Size-Based Policies | 482 |
| 30.1 | Processor-Sharing (PS) | 482 |
| 30.1.1 | Motivation behind PS | 482 |
| 30.1.2 | Ages of Jobs in the M/G/1/PS System | 483 |
| 30.1.3 | Response Time as a Function of Job Size | 484 |
| 30.1.4 | Intuition for PS Results | 487 |
| 30.1.5 | Implications of PS Results for Understanding FCFS | 487 |
| 30.2 | Preemptive-LCFS | 488 |
| 30.3 | FB Scheduling | 490 |
| 30.4 | Readings | 495 |
| 30.5 | Exercises | 496 |
| 31 | Scheduling: Non-Preemptive, Size-Based Policies | 499 |
| 31.1 | Priority Queueing | 499 |
| 31.2 | Non-Preemptive Priority | 501 |
| 31.3 | Shortest-Job-First (SJF) | 504 |
| 31.4 | The Problem with Non-Preemptive Policies | 506 |
| 31.5 | Exercises | 507 |
| 32 | Scheduling: Preemptive, Size-Based Policies | 508 |
| 32.1 | Motivation | 508 |
| 32.2 | Preemptive Priority Queueing | 508 |
| 32.3 | Preemptive-Shortest-Job-First (PSJF) | 512 |
| 32.4 | Transform Analysis of PSJF | 514 |
| 32.5 | Exercises | 516 |
| 33 | Scheduling: SRPT and Fairness | 518 |
| 33.1 | Shortest-Remaining-Processing-Time (SRPT) | 518 |
| 33.2 | Precise Derivation of SRPT Waiting Time* | 521 |

| | | |
|---------------------|---------------------------------------|-----|
| 33.3 | Comparisons with Other Policies | 523 |
| 33.3.1 | Comparison with PSJF | 523 |
| 33.3.2 | SRPT versus FB | 523 |
| 33.3.3 | Comparison of All Scheduling Policies | 524 |
| 33.4 | Fairness of SRPT | 525 |
| 33.5 | Readings | 529 |
| <i>Bibliography</i> | | 531 |
| <i>Index</i> | | 541 |