# CONTENTS


page

*Foreword by John Miles* xiii

*Preface* xix

1 INTRODUCTION

**1** Introduction 1

**2** Mechanisms of instability 4

**3** Fundamental concepts of hydrodynamic stability 8

**4** Kelvin–Helmholtz instability 14

**5** Break-up of a liquid jet in air 22

Problems for chapter 1 27

2 THERMAL INSTABILITY

**6** Introduction 32

**7** The equations of motion 34

The exact equations, 34; The Boussinesq equations, 35

**8** The stability problem 37

The linearized equations, 37; The boundary conditions, 40; Normal modes, 42

**9** General stability characteristics 44

Exchange of stabilities, 44; A variational principle, 45

**10** Particular stability characteristics 50

Free–free boundaries, 50; Rigid–rigid boundaries, 51; free–rigid boundaries, 52

**11** The cells 52

**12** Experimental results 59

**13** Some applications 62

Problems for chapter 2 63

## 3 CENTRIFUGAL INSTABILITY


**14** Introduction 69
**15** Instability of an inviscid fluid 71
Three-dimensional disturbances, 73; Axisymmetric disturbances, 77; Two-dimensional disturbances, 80
**16** Instability of Couette flow of an inviscid fluid 82
**17** The Taylor problem 88
Axisymmetric disturbances, 90; Two-dimensional disturbances, 103; Three-dimensional disturbances, 104; Some experimental results, 104
**18** The Dean problem 108
The Dean problem, 108; The Taylor–Dean problem, 113
**19** The Görtler problem 116
Problems for chapter 3 121


## 4 PARALLEL SHEAR FLOWS


**20** Introduction 124


### The inviscid theory


**21** The governing equations 126
**22** General criteria for instability 131
**23** Flows with piecewise-linear velocity profiles 144
Unbounded vortex sheet, 145; Unbounded shear layer, 146; Bounded shear layer, 147
**24** The initial-value problem 147


### The viscous theory


**25** The governing equations 153
**26** The eigenvalue spectrum for small Reynolds numbers 158
A perturbation expansion, 159; Sufficient conditions for stability, 161
**27** Heuristic methods of approximation 164
The reduced equation and the inviscid approximations, 165; The boundary-layer approximation near a rigid wall, 167; The WKBJ approximations, 167; The local turning-point approximations,

171; The truncated equation and Tollmien's improved viscous approximations, 175; The viscous correction to the singular inviscid solution, 177

**28** Approximations to the eigenvalue relation 180

Symmetrical flows in a channel, 181; Flows of the boundary-layer type, 183; The boundary-layer approximation to $\phi_3(z)$, 184; The WKBJ approximation to $\phi_3(z)$, 185; The local turning-point approximation to $\phi_3(z)$, 188; Tollmien's improved approximation to $\phi_3(z)$, 191

**29** The long-wave approximation for unbounded flows 196

**30** Numerical methods of solution 202

Expansions in orthogonal functions, 203; Finite-difference methods, 206; Initial-value methods (shooting), 207

**31** Stability characteristics of various basic flows 211

Plane Couette flow, 212; Poiseuille flow in a circular pipe, 216; Plane Poiseuille flow, 221; Combined plane Couette and plane Poiseuille flow, 223; The Blasius boundary-layer profile, 224; The asymptotic suction boundary-layer profile, 227; Boundary layers at separation, 229; The Falkner–Skan profiles, 231; The Bickley jet, 233; The hyperbolic-tangent shear layer, 237

**32** Experimental results 239

Problems for chapter 4 245

5 UNIFORM ASYMPTOTIC APPROXIMATIONS

**33** Introduction 251

Plane Couette flow

**34** The integral representations of the solutions 256

**35** The differential equation method 263

General velocity profiles

**36** A preliminary transformation 265

**37** The inner and outer expansions 267
The inner expansions, 268; The outer expansions, 271; The central matching problem, 276; Composite approximations, 278
**38** Uniform approximations 280
The solution of well-balanced type, 280; The solutions of balanced type, 280; The solutions of dominant-recessive type, 283
**39** A comparison with Lin's theory 285
**40** Preliminary simplification of the eigenvalue relation 290
**41** The uniform approximation to the eigenvalue relation 295
A computational form of the first approximation to the eigenvalue relation, 299; Results for plane Poiseuille flow, 301
**42** A comparision with the heuristic approximations to the eigenvalue relation 305
The local turning-point approximation to $\phi_3(z)$, 305; Tollmien's improved approximation to $\phi_3(z)$, 306; The uniform approximation to $\phi_3(z)$ based on the truncated equation, 308; The uniform approximation to $\phi_3(z)$ based on the Orr–Sommerfeld equation, 309
**43** A numerical treatment of the Orr–Sommerfeld problem using compound matrices 311
Symmetrical flows in a channel, 315; Boundary-layer flows, 316
Problems for chapter 5 317

## 6 ADDITIONAL TOPICS IN LINEAR STABILITY THEORY

**44** Instability of parallel flow of a stratified fluid 320
Introduction, 320; Internal gravity waves and Rayleigh–Taylor instability, 324; Kelvin–Helmholtz instability, 325
**45** Baroclinic instability 333
**46** Instability of the pinch 339
**47** Development of linear instability in time and space 345
Initial-value problems, 345; Spatially growing modes, 349

**48** Instability of unsteady flows 353
Introduction, 353; Instability of periodic flows, 354; Instability of other unsteady basic flows, 361
Problems for chapter 6 363

7 NONLINEAR STABILITY

**49** Introduction 370
Landau's theory, 370; Discussion, 376
**50** The derivation of ordinary differential systems governing stability 380
**51** Resonant wave interactions 387
Internal resonance of a double pendulum, 387; Resonant wave interactions, 392
**52** Fundamental concepts of nonlinear stability 398
Introduction to ordinary differential equations, 398; Introduction to bifurcation theory, 402; Structural stability, 407; Spatial development of nonlinear stability, 416; Critical layers in parallel flow, 420
**53** Additional fundamental concepts of nonlinear stability 423
The energy method, 424; Maximum and minimum energy in vortex motion, 432; Application of boundary-layer theory to cellular instability, 434
**54** Some applications of the nonlinear theory 435
Bénard convection, 435; Couette flow, 442; Parallel shear flows, 450
Problems for chapter 7 458

APPENDIX. A CLASS OF GENERALIZED AIRY FUNCTIONS

**A1** The Airy functions $A_k(z)$ 465
**A2** The functions $A_k(z, p)$, $B_0(z, p)$ and $B_k(z, p)$ 466
**A3** The functions $A_k(z, p, q)$ and $B_k(z, p, q)$ 472
**A4** The zeros of $A_1(z, p)$ 477
Addendum: Weakly non-parallel theories for the Blasius boundary layer 479
*Solutions* 481
*Bibliography and author index* 559
*Motion picture index* 595
*Subject index* 597