

TABLE OF CONTENTS

Preface	v
---------------	---

Chapter 1. BASIC MARCHING METHODS FOR 2D ELLIPTIC PROBLEMS

1.1 Introduction	1
1.1.1 The Impact of Direct Methods	1
1.1.2 Direct Marching Methods	2
1.1.3 History of Marching Methods	2
1.2.1 The Marching Method in 1D	4
1.2.2 The Reference 2D Problem	6
1.2.3 Operation Counts as an Index of Merit	8
1.2.4 Operation Counts for the Reference 2D Problem	9
1.2.5 Error Propagation Characteristics for the Reference 2D Problem	13
1.2.6 Gradient, Mixed, and Periodic Boundary Conditions	16
1.2.7 Irregular Mesh and Variable Coefficient Poisson Equations	19
1.2.8 Irregular Geometries	20
1.3 Other Second-Order Elliptic Equations	20
1.3.1 Advection-Diffusion Equations	20
1.3.2 Upwind Differences	22
1.3.3 Turbulence Terms	24
1.3.4 Fibonacci Scale	24
1.3.5 Helmholtz Terms	26
1.3.6 Cross Derivatives	27
1.3.7 Gradient Boundary Conditions and Cross Derivatives	29
1.3.8 Interior Flux Boundaries	31
1.4 Closing Note for Chapter 1	33
References for Chapter 1	33

Chapter 2. HIGH-ORDER EQUATIONS

2.1 Introduction	37
2.2 High-Order Accuracy Operators	37
2.3 Higher-Order Accurate Solutions by Deferred Corrections	38
2.4 Higher-Order Elliptic Equations	39
2.5 Operation Counts for Higher-Order Systems	40
2.6 Finite Element Equations	43
References for Chapter 2	44

Chapter 3. EXTENDING THE MESH SIZE: DOMAIN DECOMPOSITION

3.1 Introduction	45
3.2 Mesh Doubling by Two-Directional Marching	46
3.3 Multiple Marching	48
3.4 Patching	50
3.5 Influence Extending	51
3.6 Other Direct Methods for Extending the Mesh Size	54
3.7 Lower Accuracy Stencils Plus Iteration	55
3.8 Iterative Coupling for Subregions	56
3.8.1 Preconditioning	56
3.8.2 Block Iterative Relaxation	57
3.8.3 Schwarz Alternating Procedure	58
3.9 Higher Precision Arithmetic: Applications on Workstations and Virtual Parallel Networks	60
References for Chapter 3	62

Chapter 4. BANDED APPROXIMATIONS TO INFLUENCE MATRICES

4.1 Introduction	67
4.2 Banded Approximation to C	67
4.3 Operation Count and Storage for Banded CB	69
4.4 Intrinsic Storage: Data Compression for Massively Parallel Computers	71
4.5 Banded Approximation to $\hat{C}$	73
References for Chapter 4	74

Chapter 5. MARCHING METHODS IN 3D

5.1 Introduction	77
5.2 Simple 3D Marching	77
5.3 Error Propagation Characteristics for the 3D EVP	78
5.4 Operation Count and Storage Penalty for the 3D EVP Method	79
5.5 Banded Approximations in 3D	80
5.6 Operation Count for Banded Approximation in 3D	82
5.7 Additional Terms in the 3D Marching Method	83
5.8 3D EVP-FFT Method	84
5.9 Error Propagation Characteristics for 3D EVP-FFT Method	84
5.10 Operation Count and Storage Penalty for the 3D EVP-FFT Method	84
5.11 Accuracy and Additional Terms in 3D EVP-FFT Method	86
5.12 N -Plane Relaxation Within Multigrid and Domain Decomposition Methods ..	87
References for Chapter 5	88

Chapter 6. PERFORMANCE OF THE 2D GEM CODE

6.1 Introduction 91

6.2 Uses and Users 91

6.3 Overview of the GEM Codes 92

6.4 Problem Description in the Basic GEM Code 92

6.5 Tests of the Basic GEM Code 95

6.6 The Stabilizing Codes GEMPAT2 and GEMPAT4 96

6.7 Timing Tests of the Stabilized Codes 97

6.8 Representative Accuracy Testing 100

6.9 Conclusions 102

References for Chapter 6 103

Chapter 7. VECTORIZATION AND PARALLELIZATION

7.1 Introduction 105

7.2 Vectorizing the Tridiagonal Algorithm and the 9-Point March 105

7.3 Vectorizing the 5-Point March 106

7.4 Timing and Accuracy for the Vectorized Marches 106

7.5 Efficiencies 108

7.6 Multiprocessor Architectures 110

References for Chapter 7 111

**Chapter 8. SEMIDIRECT METHODS FOR NONLINEAR EQUATIONS
OF FLUID DYNAMICS**

8.1 Introduction: Time-Dependent Calculations vs. Semidirect Methods 113

8.2 Burgers Equation by Time Accurate Methods 114

8.3 Basic Idea of Semidirect Methods 115

8.4 Burgers Equation by Picard Semidirect Iteration 115

8.5 Further Discussion of the Picard Semidirect Iteration 117

8.6 Genesis of Semidirect Methods 117

8.7 NOS Method 119

8.8 LAD Method 120

8.9 Performance of NOS and LAD on the Driven Cavity Problem 120

8.10 Relative Importance of Lagging Boundary Conditions 126

8.11 Performance of LAD and NOS on a Flow-Through Problem 127

8.12 Optimum Relaxation Factor and Convergence for Large Problems 128

8.13 Choice between LAD and NOS 133

8.14 Split NOS Method 134

8.15 A Better Boundary Condition on Wall Vorticity 136

 8.15.1 The Trouble with the Conventional Methods for Wall Vorticity 136

 8.15.2 Israeli-Dorodnicyn Method for Wall Vorticity 137

 8.15.3 Analytical Prediction of Optimum g 137

 8.15.4 Performance of Israeli-Dorodnicyn Method 138

8.16 Dorodnicyn-Meller Method 139

8.17 Viscous Flows in Alternate Variables 139

8.18 BID Method 139

 8.18.1 BID Iteration 139

8.18.2	BID Boundary Conditions for the Driven Cavity Problem	141
8.18.3	BID Performance for the Driven Cavity	142
8.18.4	BID Performance for Flow-Through Problems	144
8.19	FOD and Coupled Systems Solvers	145
8.20	Other Applications and Non-Time-Like Methods	146
8.21	Remarks on Soution Uniqueness	147
8.22	Remarks on Semidirect Methods within Domain Decomposition	148
	References for Chapter 8	148

Chapter 9. COMPARISON TO MULTIGRID METHODS

9.1	Introduction	153
9.2	Definition of the Methods	153
9.3	Treatment of Nonlinearities	153
9.4	Speed and Accuracy	154
9.5	Grid Sensitivity and Word Length Sensitivity	155
9.6	Directionality	155
9.7	Storage Penalty	156
9.8	Dimensionality	156
9.9	Work Estimates	157
9.10	Boundary Conditions	157
9.11	General Coefficient Problems	157
9.12	Grid Transformations	157
9.13	Irregular Logical-Space Geometry	157
9.14	Higher Order Systems	157
9.15	Higher Order Accuracy Equations	158
9.16	Finite Element Equations	158
9.17	Use in Time Dependent Problems	158
9.18	Cell Reynolds Number Difficulties	158
9.19	Virtual Problems	158
9.20	MLAT and other Grid Adaptation	159
9.21	Vectorization, Parallelization, and Convergence Testing	159
9.22	Simplicity, Modularity, and Robustness	160
9.23	Summary	160
	References for Chapter 9	161

APPENDIX A. MARCHING SCHEMES AND ERROR PROPAGATION FOR VARIOUS DISCRETE LAPLACIANS

A.1	Introduction	163
A.2	Standard 5-Point Laplacian	163
A.3	Uneven Mesh	164
A.4	Other Elliptic Operators	164
A.5	Other Analogs for the Laplacian	168
	References for Appendix A	172

**APPENDIX B. TRIDIAGONAL ALGORITHM FOR
PERIODIC BOUNDARY CONDITIONS**

B.1 Introduction 173
B.2 Problem Statement 173
B.3 "Algorithm 4" 174
B.4 The Influence Coefficient "Algorithm 5" 174
B.5 Completed "Algorithm 4" 175
B.6 Operation Counts and Other Comparisons 176
References for Appendix B 178

APPENDIX C. GAUSS ELIMINATION AS A DIRECT SOLVER

C.1 Introduction 179
C.2 Round-Off Error 179
C.3 Speed 179
C.4 Storage Penalty 179
C.5 Operation Count and Storage Penalty for 2D 180
C.6 Operation Count and Storage Penalty for 3D 182
C.7 Conclusions 184
References for Appendix C 184

SUBJECT INDEX 185