Contents

1 Theoretical Basis of the Structural Modeling Method 1
1.1 Review of References 1
1.1.1 Discrete and Continuum Models of Solids: A Brief Historical Review 2
1.1.2 Development of Models of Microstructured Solids with Account of Particle Rotation 5
1.1.3 Experimental Research of Dynamic Properties of Microstructured Media 6
1.2 Methods of Description of Different Scale Levels 8
1.3 Limits of Applicability of the Classical Mechanics Laws to Modeling of Generalized Continua 13
1.3.1 Quantum and Classical Descriptions of Microparticles 13
1.3.2 The Uncertainty Relation 15
1.3.3 A Microparticle as a Localized Wave Packet 16
1.3.4 The Conformity Principle 18
1.4 Principles of the Structural Modeling Method 20
1.5 Conclusions 25
References 25

2 A 2D Lattice with Dense Packing of the Particles 35
2.1 The Discrete Model for a Hexagonal Lattice Consisting of Round Particles 35
2.2 The Continual Approximation 39
2.3 Influence of Microstructure on Acoustic Properties of a Medium 41
2.4 Dispersion Properties of Normal Waves 43
2.4.1 Dispersion Properties of the Discrete Model 43
2.4.2 Dispersion Properties of the Continual Model 48
2.5 Conclusions 51
References 52

3 A Two-Dimensional Lattice with Non-dense Packing of Particles . . . 55
3.1 The Discrete Model for an Anisotropic Medium Consisting of Ellipse-Shaped Particles . . . 55
3.2 The Continuum Approximation . . . 60
3.2.1 Dependence of the Anisotropy of the Medium on Its Microstructure . . . 61
3.2.2 A Square Lattice of Round Particles . . . 64
3.2.3 A Chain of Ellipse-Shaped Particles . . . 64
3.3 Influence of Microstructure on Acoustic Properties of the Medium . . . 65
3.3.1 Dependence of the Elastic and Rotational Wave Velocities on the Shape of the Particles in the 1D Lattice . . . 65
3.3.2 Dependence of the Acoustic Characteristics of the 2D Anisotropic Medium on the Microstructure Parameters . . . 67
3.4 Dispersion Properties of Normal Waves . . . 69
3.4.1 Dispersion Properties of the Discrete Model . . . 69
3.4.2 Dispersion Properties of the Continual Model . . . 75
3.5 Conclusions . . . 79
References . . . 80

4 Application of the 2D Models of Media with Dense and Non-dense Packing of the Particles for Solving the Parametric Identification Problems . . . 83
4.1 Reduced (Gradient) Models of the Theory of Elasticity . . . 83
4.2 Problems of the Material Identification . . . 87
4.2.1 Identification of the Medium with Hexagonal Symmetry . . . 88
4.2.2 Identification of the Medium with Cubic Symmetry . . . 91
4.3 Comparison with the Cosserat Continuum Theory . . . 95
4.4 Influence of the Microstructure on the Poisson's Ratio of an Isotropic Medium . . . 97
4.5 Influence of the Microstructure on the Poisson's Ratios of the Anisotropic Medium . . . 101
4.6 Conclusions . . . 104
References . . . 105

5 Nonlinear Models of Microstructured Media . . . 109
5.1 A Rectangular Lattice Consisting of Ellipse-Shaped Particles . . . 109
5.2 Estimation of the Nonlinearity Coefficients of the Mathematical Model of the Square Lattice of Round Particles . . . 115
5.3 The Square Lattice of Nanotubes . . . 119
5.3.1 The Discrete Model . . . 120
5.3.2 The Continual Approximation . . . 122

5.3.3 Relationships Between the Macroparameters of the Material and the Parameters of Its Inner Structure 124
5.4 Conclusions 126
References 126

6 A Cubic Lattice of Spherical Particles 129
6.1 A Discrete 3D Model of a Crystalline Medium of Spherical Particles 129
6.2 Nonlinear Model of a One-Layer Medium of Spherical Particles 133
6.2.1 The Continuum Approximation 133
6.2.2 Dependency of the Macroparameters of a One-Layer Medium on the Parameters of Its Microstructure 135
6.2.3 3D Model of a Crystalline Medium of Spherical Particles 137
6.2.4 Continuum Approximation 137
6.2.5 Dependence of the Macroparameters of the 3D Medium on the Parameters of Its Microstructure 138
6.2.6 Comparison of the Proposed Model with the 3D Cosserat Continuum 140
6.3 Conclusions 141
References 144

7 Propagation and Interaction of Nonlinear Waves in Generalized Continua 147
7.1 Localized Strain Waves in a 2D Crystalline Medium with Non-dense Packing of the Particles 147
7.2 A 1D Medium Consisting of Ellipse-Shaped Particles and with Internal Stresses 152
7.2.1 Mechanical Model of a 1D Medium with Internal Stresses 153
7.2.2 Equations of the Gradient Theory of Elasticity for a 1D Medium with Internal Stresses 156
7.3 Self-modulation of Shear Strain Waves Propagating in a 1D Granular Medium 158
7.3.1 The Modulation Instability Areas 158
7.3.2 Forms of Wave Packets in the Case of the Modulation Instability 161
7.4 Nonlinear Longitudinal Waves in a Rod Made of an Auxetic Material 164
7.4.1 The Linear Mathematical Model. Dispersion Properties . . . 164
7.4.2 The Nonlinear Mathematical Model. Stationary Strain Waves 166
7.4.3 Numerical Simulation of Soliton Interactions 175

7.5 Application of an Alternative Continualization Method for Analysis of Nonlinear Localized Waves in a Gradient-Elastic Medium 180
7.5.1 One-Dimensional Model of a Nonlinear Gradient-Elastic Continuum 181
7.5.2 Nonlinear Strain Waves 183
7.6 Conclusions 187
References 189

Correction to: Structural Modeling of Metamaterials C1

Discussion of the Results 195

Appendix A: Expressions for Elongation of the Springs in the Hexagonal Lattice 201

Appendix B: Expressions for Elongation of the Springs in the Rectangular Lattice 203

References 207