

---

# Contents

---

Preface	xi
<b>SECTION I Groups</b>	
<b>CHAPTER 1 ■ Background Material</b>	<b>3</b>
1.1 EQUIVALENCE RELATIONS	3
1.2 FUNCTIONS	9
1.3 BASIC NUMBER THEORY	16
1.4 MODULO ARITHMETIC	20
<b>CHAPTER 2 ■ Basic Group Theory</b>	<b>24</b>
2.1 DEFINITIONS AND EXAMPLES	24
2.1.1 Groups of Small Order	27
2.1.2 Group Exponentiation	30
2.2 SUBGROUPS	32
2.3 CYCLIC GROUPS	36
2.4 PERMUTATION GROUPS	41
2.5 PRODUCTS OF GROUPS	50
2.6 HOMOMORPHISMS	54
2.7 ISOMORPHIC GROUPS	60
2.8 COSETS OF A GROUP	64
2.9 FACTOR GROUPS AND NORMAL SUBGROUPS	69
2.9.1 Semidirect Products	76
2.10 NORMAL AND SIMPLE GROUPS	81
2.11 THE GROUP ISOMORPHISM THEOREMS	83
<b>CHAPTER 3 ■ Simple Groups</b>	<b>89</b>
3.1 THE ALTERNATING GROUP	89

3.2	THE PROJECTIVE LINEAR GROUPS	92
CHAPTER	4 ■ Group Action	102
4.1	GROUP ACTION ON A SET	102
4.2	BURNSIDE'S LEMMA	108
4.3	POLYA'S FORMULA	111
4.4	SOME CONSEQUENCES OF GROUP ACTION	117
4.5	SYLOW THEORY	122
4.6	CLASSIFYING FINITE GROUPS WITH SYLOW THEORY	126
4.7	FINITE ABELIAN GROUPS	131
CHAPTER	5 ■ Group Presentation and Representations	136
5.1	FREE GROUPS	136
5.2	GROUP PRESENTATIONS	140
5.3	GROUP REPRESENTATION	144
CHAPTER	6 ■ Solvable and Nilpotent Groups	148
6.1	SOME RELEVANT SUBGROUPS	148
6.2	SERIES OF GROUPS	154
6.3	SOLVABLE AND NILPOTENT GROUPS	158
SECTION II Rings and Fields		
CHAPTER	7 ■ Ring Theory	165
7.1	DEFINITION AND EXAMPLES	165
7.2	INTEGRAL DOMAINS	169
7.3	THE QUATERNIONS	172
7.4	RING HOMOMORPHISMS	175
7.5	FACTOR RINGS AND IDEALS	178
7.6	QUOTIENT FIELD OF AN INTEGRAL DOMAIN	183
7.7	CHARACTERISTIC OF A RING	186
7.8	THE RING OF POLYNOMIALS	188
7.9	SPECIAL IDEALS	192

CHAPTER	8 ■ Integral Domain Theory	196
8.1	EUCLIDEAN AND PRINCIPAL IDEAL DOMAINS	196
8.2	UNIQUE FACTORIZATION DOMAINS	203
8.3	ONE PARTICULAR INTEGRAL DOMAIN	206
8.4	POLYNOMIALS OVER A UFD	209
CHAPTER	9 ■ Field Theory	216
9.1	REVIEW AND ALGEBRAICITY	216
9.2	VECTOR SPACES & EXTENSION FIELDS	220
9.3	GEOMETRIC CONSTRUCTIONS	228
9.3.1	Famous Impossibilities	232
9.4	ALGEBRAIC EXTENSION & CLOSURE	234
9.5	EXISTENCE THEOREMS	238
9.6	FINITE FIELDS	240
CHAPTER	10 ■ Galois Theory	246
10.1	FIELD HOMOMORPHISMS	246
10.2	COMPUTING GALOIS GROUPS	249
10.3	APPLICATIONS OF ZORN'S LEMMA	252
10.4	TWO IMPORTANT THEOREMS	256
10.5	SEPARABLE DEGREE	259
10.6	GALOIS EXTENSIONS	261
10.7	SOME PRELIMINARY THEOREMS	264
10.8	THE FUNDAMENTAL THEOREM OF GALOIS THEORY	267
10.9	SOLVABLE GROUP ESSENTIALS	271
10.10	SOLVABILITY BY RADICALS	275
References		285
Index		287