

CONTENTS

Chapter 1	Fundamental Notions	1
1	Sets	2
1.1	A set is a collection of objects	2
1.2	Events as sets	4
1.3	Sets and the vernacular of events	6
1.4	Special kinds of sets	8
2	Probability	14
2.1	Intuitive background	14
2.2	The probability function	15
2.3	The experiment	16
2.4	Equally likely events	17
2.5	Counting	19
3	Applications	35
3.1	Comments on mathematical models	35
3.2	Acceptance sampling	36
3.3	The occupancy problem and statistical mechanics	38
4	Additional problems, theoretical and applied	41
4.1	Practice problems	42
Chapter 2	Independence	45
1	Independence	45
1.1	Conditional probability and independent events	45
1.2	Mutual independence	51
1.3	Independent trials	54
2	Mathematical model of coin tossing	60
2.1	The development of the model	60
2.2	Applications	61
2.3	The binomial	65
3	Applications: Entropy	65
3.1	Entropy and information	71
3.2	Entropy and statistical mechanics	74
4	Applications to genetics and physical anthropology	76
5	Additional problems	80
5.1	Practice problems	84

Chapter 3	Markov chains: Matrices	86
1	Markov chains	86
1.1	Introduction	86
1.2	Limiting distribution	93
2	Matrices.	97
2.1	Definitions	98
2.2	Operations	98
3	Markov matrices	100
4	Applications	103
4.1	Entropy and information	103
4.2	Comments on social mobility models	105
4.3	Acceptance sampling	105
4.4	Menu planning	106
5	Additional problems	108
Chapter 4	Countable Sample Spaces	111
1	Countable sets	111
1.1	Events and countable sets	115
2	The probability measure and the extension problem	116
2.1	The completely additive property	116
2.2	Examples of the extension problem	120
3	Examples of countable sample spaces	123
3.1	Poisson distribution: catastrophic events	123
3.2	Applications of the Poisson-type model	130
3.3	Borel-Cantelli lemmas	134
4	Additional problems	138
4.1	Practice problems	139
Chapter 5	Expectation: Random Variables	142
1	Definitions	142
2	Sums of random variables.	146
2.1	Independent random variables	151
2.2	Products of random variables	155
3	Variance	156
4	Generating functions	160
4.1	Probability generating function (pgf)	161
4.2	Moment generating function (mgf)	164
4.3	Applications of generating functions	168
5	Applications	174
5.1	Entropy	174
5.2	Entropy and information—a general point of view	177
5.3	Statistical mechanics	178
5.4	Acceptance sampling	180
5.5	Recurrent events; coin-tossing model	182
6	Additional problems	185
6.1	Practice problems	186

Chapter 6	Infinite Sample Spaces; Random Variables; Distributions	189
1	Noncountable sample spaces; the extension problem	189
1.1	Noncountable spaces	189
1.2	The extension problem	190
2	Examples and discussions	197
2.1	Borel sets, nonmeasurable sets	197
2.2	Absorbing barrier problem	199
2.3	Appendix: B' is a nonobservable event	203
3	Random variables	205
3.1	Definitions	205
3.2	Fundamental theorems	207
3.3	Independent random variables	208
3.4	Random variables and independent trials	209
3.5	Sequences of random variables	211
4	The distribution function	213
4.1	Definition	213
4.2	The distribution function and probability measure	216
4.3	Multidimensional distribution functions	219
5	Applications	222
5.1	Stochastic processes	222
5.2	Queuing theory	222
5.3	Reliability and failure laws	224
6	Additional problems	227
6.1	Practice problems	227
Chapter 7	Integration and Expectation	231
1	Introduction: The Riemann integral and the expected value	231
1.1	Moments and moment generating functions	245
1.2	Expectation of products: covariance	247
2	Failure laws and aetiological considerations in medical research	249
2.1	Appendix: The Weibull distribution as a "failure law" in autoimmune diseases	249
3	Conditional densities and conditional expectations	250
3.1	Conditional probability	250
3.2	Conditional density	252
3.3	Conditional expectation	255
4	Riemann-Stieltjes integral	257
5	The Lebesgue integral (finite measure)	261
5.1	Formal definition	263
5.2	Fundamental theorems	265
6	Practice problems	268
Chapter 8	Sums of Random Variables	270
1	Sums when the densities are Riemann integrable	271
1.1	Direct integration	271
1.2	Transformation of variables	274

2	Sums of independent random variables	278
2.1	Convolution of densities	278
2.2	Convolution of distribution functions	280
2.3	The characteristic function	282
2.4	The characteristic function (c.f.) as a moment generating function	288
3	Additional problems	290

Chapter 9	Statistics	293
1	Introduction	293
2	Notation	294
3	Point estimation	295
3.1	Introduction	295
3.2	Maximum likelihood estimate (MLE)	297
3.3	Efficiency	300
4	Interval estimation	308
4.1	Introduction	308
4.2	Confidence intervals	308
5	Hypothesis testing	311
5.1	Simple hypotheses	314
5.2	Important special case for normal distributions	320
6	A central limit theorem and applications	321
6.1	Acceptance sampling: control charts	323
6.2	Statistics and the law	324
7	Applications	328
7.1	Monte Carlo methods	328
7.2	On the use of computers in statistical analysis	330
8	Additional problems	331
8.1	Practice problems	332

Chapter 10	Limit Theorems	334
1	Introduction	334
2	Types of convergence	336
2.1	Convergence in probability and strong convergence	336
2.2	Convergence in distribution	342
2.3	Mean convergence	349
2.4	The central limit problem	351
3	Application: Stochastic approximation method of Robbins-Monroe	356
4	Additional problems	359
4.1	Practice problems	360

Bibliography	362
Appendix	
Table 1 Cumulative normal distribution	365
Table 2 Critical values for student's t -distributions	368
Table 3 Critical values for the chi-square distribution	371
Selected Answers	374
Index	397