
CONTENTS

Chapter I INTRODUCTION TO THE DERIVATIVE

1 Two Problems of Optimization	3
2 The Problem of Tangents	12
3 Descartes' Method of Equal Roots	21
4 Fermat's Method of Limits	32
5 Solution of the Optimization Problems	41
6 Some Rules for Slope-Predicting Formulas	48
7 The Rocket Problem	58
8 Independent and Dependent Variables	66
9 Domain and Range of a Function	74
10 Local and Global Extreme Values	85
11 Instantaneous Rates of Change	93
12 Further Differentiation Rules	101
13 Leibnitz's Notation and Non-Time Rates	109
14 Continuation of the Rocket Problem	117
15 The Concept of Continuity	126
16 Derivatives of the Circular Functions	138
17 Antiderivatives	147
*18 An Application in Optics	155
*19 An Application in Business	162
*20 An Application in Numerical Analysis	170

Chapter II INTRODUCTION TO THE INTEGRAL

21 Buffon's Needle Problem	181
22 Eudoxus' Method of Exhaustion	187
23 Summation and Sigma Notation	198
24 Curvilinear Areas and the Integral Sign	208
25 Some General Area-Predicting Formulas	216
26 Barrow's Theorem on Evaluating Integrals	227
27 The Integral as a Limit of Riemann Sums	240
28 Volumes of Geometric Solids	251
29 The Calculation of Work	261
30 An Example with the Ellipse	268
*31 Hydrostatic Pressure	276
32 Infinite Sums and Improper Integrals	282

*optional lesson

Chapter III CALCULUS WITH COMPOSITE FUNCTIONS

33	Concavity and the Second-Derivative Test	292
34	Linearization and Approximation	301
35	A Proof of the Error Formula	310
36	The Mean-Value Principle	318
37	Composite Functions and the Chain Rule	325
38	Problems Involving Related Rates	333
39	Curves Described with Parametric Equations	340
40	Cauchy's Formula and L'Hospital's Rule	349
41	Differentials and Implicit Differentiation	356
42	Problems involving Auxiliary Variables	364
43	The Average Value of a Continuous Variable	372
44	The Change of Variables Technique	379
45	Volumes by the Method of Shells	387
46	The Indefinite Integral	394
47	The Method of Integration by Parts	402
*48	The Binomial Theorem and Wallis' Formula	408

Chapter IV INTEGRATION OF ADDITIONAL ELEMENTARY FUNCTIONS

49	A Search for Some Missing Antiderivatives	416
50	Approximating Integrals by the Trapezoidal Rule	424
*51	Proof of the Trapezoidal Rule's Error Formula	432
52	The Logarithmic Laws and Their Application	437
53	Integrations Using the Natural Logarithm Function	444
54	Introduction to the Number e	450
55	The Series of Reciprocal Factorials	458
56	Rational and Irrational Numbers	463
57	Proof that e is Irrational	468
*58	Proof that π is Irrational	472
59	The Concept of Inverse Functions	479
60	Some Noninvertible Functions	489
61	Inverses for the Trigonometric Functions	498
62	The Exponential Function and its Properties	506
*63	A Different Definition of Irrational Powers?	513
64	Logarithms to Other Bases	523
65	Arc Length and Trigonometric Substitution	529
*66	Surface Area and Completing the Square	538
67	The Method of Partial Fractions	546
68	Extending the Method of Partial Fractions	551
69	Introduction to Differential Equations	556

*optional lesson

70 The Separation of Variables Technique	563
71 The Study of Population Growth	570
72 The Hanging Cable Problem	578

Appendices

I Basic Ideas of Analytic Geometry	585
II Basic Ideas of Plane Geometry	599
III Tables	611
Answers to the Exercises	614
Index	635