

Contents

1	Preliminaries	1
1.1	Introduction	2
1.1.1	Mathematical Modeling	3
1.1.2	Patterns in Numbers	6
1.1.3	Patterns in Word Problems	11
1.1.4	Problems with Hidden Patterns	11
1.1.5	Problems Where All the Needed Information Are Not Provided	12
1.2	Set and Its Basic Properties	14
1.2.1	The Classic Set Theory	17
1.2.2	Zermelo and Fraenkel (ZFC) Axiomatic Set Theory	21
1.2.3	Fuzzy Set Theory	26
1.3	Functions of One Real Variable	31
1.3.1	Basic Concepts	31
1.3.2	Inverse of a Function of One Real Variable	34
1.3.3	Differentiable Functions of One Real Variable	37
1.3.4	Some Properties of Differentiable Functions	40
1.3.5	Convex and Convey Functions	40
1.3.6	Tangent Line to a Function of One Real Variable	43
1.3.7	The Chain Rule for Functions of One Real Variable	44
1.4	Functions of More Than One Real Variable	46
1.4.1	Parametric Form of Functions with More Than One Real Variable	51
1.4.2	Limit of a Function of Two Real Variables	53
1.4.3	Differentiation of Functions of More Than One Real Variable, Partial Derivatives	56
1.4.4	Tangent Plane to Surface of a Function of Two Real Variables	58
1.4.5	The Chain Rule for Real Multivariable Functions	61
1.4.6	Extrema for Real Bivariate Functions	65

1.5	Complex Numbers and Their Properties	68
1.6	Exercises	72
2	Linear Algebra and Vector Space	77
2.1	Introduction	78
2.2	Matrix and Determinant	79
2.2.1	Basic Definitions Regarding Matrices	79
2.2.2	System of Linear Equations	81
2.2.3	Operations with Matrices	83
2.2.4	Row Reduction	84
2.2.5	Matrix Equations and the Gauss-Jordan Method	85
2.3	Determinant and Inverse of a Matrix	92
2.3.1	Determinant	92
2.3.2	Inverse of a Matrix	93
2.3.3	Cramer's Rule	102
2.4	Vectors	104
2.4.1	Arithmetic Operations on Vectors	109
2.4.2	Dot (or Scalar or Inner) Product of Vectors and Some of Its Properties	111
2.4.3	Gram-Schmidt Procedure (Constructing Orthonormal Vectors)	111
2.4.4	Some Properties of Dot (or Scalar or Inner) Product of Vectors	117
2.4.5	The Cross Product (or Vector Product) of Vectors and Some of Its Properties	118
2.4.6	Some Properties of the Cross Product	118
2.4.7	Vector Space and Linearly Independent and Dependent Vectors	120
2.4.8	Some Properties of Linearly Independent and Dependent Vectors	123
2.4.9	Eigenvalues and Eigenvectors	125
2.5	Directional Derivatives and Gradient Vectors	128
2.6	Exercises	141
3	Differential and Difference Equations	145
3.1	Introduction	146
3.2	Laplace Transform	147
3.2.1	Some Properties of Laplace Transform	151
3.3	Ordinary Differential Equations	155
3.3.1	Selected Methods for Solving Ordinary Differential Equations	160
3.4	Partial Differential Equations	182
3.4.1	Most Important, Well-Known and Uses of Partial Differential Equations in Science and Engineering	185
3.5	Solution of PDE Using the Separable Variables Method	193

3.6	Difference Equations	204
3.6.1	Basic Terms	204
3.6.2	Linear Difference Equation	206
3.6.3	Solution of Linear Difference Equations	207
3.7	Exercises	217
4	Sequences and Series	221
4.1	Introduction	222
4.2	Sequences	226
4.2.1	Infinite Sequences, General Concepts	226
4.2.2	Upper and Lower Limits	231
4.2.3	Cauchy Sequence	233
4.2.4	Suprema and Infima	234
4.3	Infinite Series	235
4.3.1	Infinite Series of Numbers, General Concepts	235
4.3.2	Selected Properties of Infinite Series	241
4.3.3	Selected Tests for Convergence of Infinite Series (Numerical or with Variables)	241
4.4	Power Series	250
4.5	Taylor and Maclaurin Series	254
4.6	Series Solution of Linear Second Order Differential Equations	263
4.7	Fourier Series	271
4.7.1	Fourier Cosine and Sine Series	279
4.8	Fourier Transform	281
4.8.1	Continuous Fourier Transform	282
4.8.2	Discrete Fourier Transform	292
4.8.3	Some Properties of a Fourier Transform	293
4.8.4	Fast Fourier Transform (FFT)	294
4.9	Solution of Partial Differential Equations Using Fourier Series and Transform	296
4.10	Exercises	302
5	Functions of Complex Variables	309
5.1	Introduction	310
5.2	Functions of a Complex Variable	311
5.3	Differentiation of a Function of a Complex Variable	313
5.4	Integration of a Function of a Complex Variable	320
5.4.1	Complex Line Integral	320
5.4.2	The Cauchy Integral Theorem	322
5.4.3	Parametrization	324
5.4.4	Harmonic Conjugate	327
5.4.5	Cauchy's Integral Formula for a Point on the Path	330
5.4.6	Derivatives of Analytic Functions (Cauchy's Integral Formula for Derivatives)	332
5.5	Complex Series	336

5.5.1	Power Series of Complex Numbers	336
5.5.2	Complex Power Series	339
5.5.3	Taylor and Maclaurin Series for Functions of a Complex Variable	343
5.5.4	Complex Generating Function	347
5.5.5	Laurent Series	351
5.5.6	Complex Residue	355
5.6	Conformal Mapping	356
5.7	Exercises	359
6	Probability	365
6.1	Introduction	366
6.2	Basic Terms and Definitions of Sets and Events	367
6.3	Basics of Probability	370
6.4	Conditional Probability	373
6.5	Discrete Random Variables and Probability Mass Functions	380
6.5.1	Discrete Random Variables	380
6.5.2	Probability Distributions or Mass Functions of Discrete Random Variables	382
6.6	Some Particular Discrete Probability Mass (Distribution) Functions (Pmf)	387
6.6.1	Bernoulli Pmf	387
6.6.2	Negative-Binomial Pmf	394
6.6.3	Geometric Mass Function	395
6.6.4	Poisson Probability Mass Function (pmf)	399
6.7	Continuous Random Variables and Probability Distribution Function (cdf)	401
6.8	Some Particular Continuous Probability Distribution Functions	403
6.8.1	Uniform or Rectangular Probability Distribution Function (Cdf)	403
6.8.2	Exponential Probability Distribution Function	405
6.8.3	Normal Distribution Function	407
6.8.4	Logistic Distribution Function	411
6.8.5	Extreme-Value Distribution Function	414
6.8.6	Some Properties of Extreme-Value	417
6.8.7	Weibull Distribution Function	418
6.9	Functions of Multiple Continuous Random Variables	420
6.10	Exercises	424
7	Statistics	431
7.1	Introduction	432
7.2	Sampling	432
7.2.1	Measures of Central Tendency	435
7.2.2	Measure of Dispersion	443
7.2.3	Expected Value and Variance of Sample Mean	448

7.3	The Law of Large Numbers	448
7.4	Estimations	450
7.4.1	Point Estimation	450
7.4.2	The Maximum Likelihood Estimator (MLE)	454
7.4.3	Interval Estimation or Confidence Interval	458
7.5	Hypothesis Testing	464
7.6	Analysis of Variance (ANOVA)	469
7.7	Linear Regression	472
7.8	Exercises	483
8	Stochastic Processes	495
8.1	Introduction	496
8.2	Stochastic (or Random) Process	496
8.3	Point Process	500
8.4	Discrete-Time Stochastic Process	504
8.4.1	Markov Chain	506
8.5	Markov Process	517
8.5.1	Examples of Queueing Theory	524
8.6	Wiener Process or Brownian Motion	537
8.6.1	Introduction	537
8.6.2	Discussion of Wiener Process or Brownian Motion	538
8.7	Exercises	540
9	Reliability	545
9.1	Introduction	546
9.1.1	What Is Reliability?	549
9.2	Reliability of Static Systems (the System Is Not Evolving in Time)	553
9.3	Discussion of Reliability of Dynamic Systems with Markov Process (Chain)	556
9.4	Survival Function	559
9.5	Reliability Test Plan	562
9.6	Estimation of Reliability Function	563
9.6.1	Estimation of Reliability Function by Logistic Distribution	564
9.6.2	Estimation of Reliability Function Using Extreme-Value Distribution	565
9.6.3	Estimation of Reliability Function with Weibull Distribution	575
9.7	Reliability Analysis of k -out-of- n : G System	577
9.7.1	Estimation of k -out-of- n System with Weibull Component	579
9.8	Exercises	581

10 Optimization	585
10.1 Introduction	586
10.2 Interpolation	589
10.3 Linear Optimization Problem or Linear Programing Problem (LPP or LP)	593
10.4 Stochastic Optimization	607
10.4.1 Comparison of Estimators	613
10.5 Non-Linear Optimization-Lagrange Multiplier	615
10.6 Exercises	622
Appendix	627
Answers for Selected Exercises	641
Bibliography	649
Index	655