

Contents

Preface xi

1 Mathematical Preliminaries and Error Analysis 1

- 1.1 Review of Calculus 2
- 1.2 Round-off Errors and Computer Arithmetic 14
- 1.3 Algorithms and Convergence 29
- 1.4 Numerical Software 38

2 Solutions of Equations in One Variable 47

- 2.1 The Bisection Method 48
- 2.2 Fixed-Point Iteration 55
- 2.3 Newton's Method and Its Extensions 66
- 2.4 Error Analysis for Iterative Methods 78
- 2.5 Accelerating Convergence 86
- 2.6 Zeros of Polynomials and Müller's Method 91
- 2.7 Numerical Software and Chapter Review 101

3 Interpolation and Polynomial Approximation 103

- 3.1 Interpolation and the Lagrange Polynomial 104
- 3.2 Data Approximation and Neville's Method 115
- 3.3 Divided Differences 122
- 3.4 Hermite Interpolation 134
- 3.5 Cubic Spline Interpolation 142
- 3.6 Parametric Curves 162
- 3.7 Numerical Software and Chapter Review 168

4 Numerical Differentiation and Integration 171

- 4.1 Numerical Differentiation 172
- 4.2 Richardson's Extrapolation 183
- 4.3 Elements of Numerical Integration 191

- 4.4 Composite Numerical Integration 202
- 4.5 Romberg Integration 211
- 4.6 Adaptive Quadrature Methods 219
- 4.7 Gaussian Quadrature 228
- 4.8 Multiple Integrals 235
- 4.9 Improper Integrals 250
- 4.10 Numerical Software and Chapter Review 256

5 Initial-Value Problems for Ordinary Differential Equations 259

- 5.1 The Elementary Theory of Initial-Value Problems 260
- 5.2 Euler's Method 266
- 5.3 Higher-Order Taylor Methods 275
- 5.4 Runge-Kutta Methods 282
- 5.5 Error Control and the Runge-Kutta-Fehlberg Method 294
- 5.6 Multistep Methods 302
- 5.7 Variable Step-Size Multistep Methods 316
- 5.8 Extrapolation Methods 323
- 5.9 Higher-Order Equations and Systems of Differential Equations 331
- 5.10 Stability 340
- 5.11 Stiff Differential Equations 349
- 5.12 Numerical Software 357

6 Direct Methods for Solving Linear Systems 361

- 6.1 Linear Systems of Equations 362
- 6.2 Pivoting Strategies 376
- 6.3 Linear Algebra and Matrix Inversion 386
- 6.4 The Determinant of a Matrix 400
- 6.5 Matrix Factorization 406
- 6.6 Special Types of Matrices 416
- 6.7 Numerical Software 433

7 Iterative Techniques in Matrix Algebra 437

- 7.1 Norms of Vectors and Matrices 438
- 7.2 Eigenvalues and Eigenvectors 450
- 7.3 The Jacobi and Gauss-Siedel Iterative Techniques 456
- 7.4 Relaxation Techniques for Solving Linear Systems 469
- 7.5 Error Bounds and Iterative Refinement 476
- 7.6 The Conjugate Gradient Method 487
- 7.7 Numerical Software 503

8 Approximation Theory 505

- 8.1 Discrete Least Squares Approximation 506
- 8.2 Orthogonal Polynomials and Least Squares Approximation 517
- 8.3 Chebyshev Polynomials and Economization of Power Series 526
- 8.4 Rational Function Approximation 535
- 8.5 Trigonometric Polynomial Approximation 545
- 8.6 Fast Fourier Transforms 555
- 8.7 Numerical Software 567

9 Approximating Eigenvalues 569

- 9.1 Linear Algebra and Eigenvalues 570
- 9.2 Orthogonal Matrices and Similarity Transformations 578
- 9.3 The Power Method 585
- 9.4 Householder's Method 602
- 9.5 The QR Algorithm 610
- 9.6 Singular Value Decomposition 624
- 9.7 Numerical Software 638

10 Numerical Solutions of Nonlinear Systems of Equations 641

- 10.1 Fixed Points for Functions of Several Variables 642
- 10.2 Newton's Method 651
- 10.3 Quasi-Newton Methods 659
- 10.4 Steepest Descent Techniques 666
- 10.5 Homotopy and Continuation Methods 674
- 10.6 Numerical Software 682

11 Boundary-Value Problems for Ordinary Differential Equations 685

- 11.1 The Linear Shooting Method 686
- 11.2 The Shooting Method for Nonlinear Problems 693
- 11.3 Finite-Difference Methods for Linear Problems 700
- 11.4 Finite-Difference Methods for Nonlinear Problems 706
- 11.5 The Rayleigh-Ritz Method 712
- 11.6 Numerical Software 728



12 Numerical Solutions to Partial Differential Equations 731

- 12.1 Elliptic Partial Differential Equations 734
- 12.2 Parabolic Partial Differential Equations 743
- 12.3 Hyperbolic Partial Differential Equations 757
- 12.4 An Introduction to the Finite-Element Method 765
- 12.5 Numerical Software 779

Bibliography 781

Answers to Selected Exercises 787

Index 889