

Contents

1. Mathematical preliminaries	1
1.1 Components of a vector	1
1.2 Dot or scalar product	1
1.3 Cross or vector product	2
1.4 Derivative of a vector	3
1.5 Results involving derivatives	3
1.6 Partial derivatives of vectors	4
1.6.1 The gradient of a scalar field	4
1.6.2 The divergence of a vector field	6
1.6.3 The Laplacian of a scalar or vector field	6
1.6.4 The curl of a vector field	7
1.6.5 Other formulae involving ∇	7
1.7 Divergence of a vector field: an application	7
1.8 Divergence or Green's theorem	9
1.9 Green's theorem in two dimensions	13
1.10 Orthogonal curvilinear coordinates	14
1.11 Gradient and Laplacian in orthogonal curvilinear coordinates	17
1.12 Integral transforms	20
1.12.1 Laplace transform	21
1.12.2 Fourier transforms	41
1.12.3 Hankel transforms	57
1.13 PROBLEM SET 1	63
2. General concepts in partial differential equations	71
2.1 Fundamental concepts	72
2.1.1 The order of a partial differential equation	73
2.1.2 The linearity of a partial differential equation	74
2.1.3 Homogeneity of a partial differential equation	76
2.2 Well-posed problems	77
2.2.1 Boundary conditions	77
2.2.2 Initial conditions	80

2.2.3	Well-posed problems.....	81
2.3	PROBLEM SET 2.....	82
3.	Partial differential equations of the first-order	87
3.1	General concepts	87
3.2	Examples involving first-order equations	90
3.3	Advective transport in reactor column	100
3.3.1	Governing equation - one dimensional case	101
3.3.2	Governing equation - generalized formulation	105
3.4	A heat exchanger problem	114
3.5	PROBLEM SET 3.....	116
4.	Partial differential equations of the second-order	121
4.1	Classification of second-order partial differential equations ..	122
4.2	Reduction to canonical forms.....	125
4.3	Applications of the procedures	132
4.4	Classification of second-order pdes for n independent variables	140
4.5	PROBLEM SET 4.....	148
5.	Laplace's equation	151
5.1	Derivation of Laplace's equation	152
5.1.1	Irrotational flow in fluid mechanics	152
5.1.2	Flow of fluids in porous media	159
5.2	Boundary conditions	166
5.2.1	Boundary conditions for fluid flow	166
5.2.2	Boundary conditions for porous media flow	167
5.2.3	Boundary conditions for heat conduction	168
5.3	Generalized results.....	169
5.4	Methods of solution of Laplace's equation.....	177
5.4.1	Direct solution procedure	178
5.4.2	Separation of variables method - Cartesian coordinates	181
5.4.3	Separation of variables method - plane polar coordinates	195
5.5	Integral transform solution of Laplace's equation.....	204
5.6	Line source within a half-plane region	210
5.7	Uniqueness theorem.....	214
5.8	A maximum principle	218
5.9	PROBLEM SET 5.....	220

6. The diffusion equation	235
6.1 Derivation of the diffusion equation	235
6.1.1 Heat conduction in solids	236
6.1.2 Pressure transients in porous media	240
6.1.3 Chemical mass transport in porous media	244
6.1.4 Drying of porous solids	248
6.1.5 Thermal oxidation of silicon	250
6.1.6 Motion of a plate on a viscous fluid	252
6.2 Initial conditions and boundary conditions	254
6.2.1 Dirichlet-type boundary condition	254
6.2.2 Neumann-type boundary conditions	255
6.2.3 Combined boundary conditions	256
6.2.4 Mixed boundary conditions	257
6.2.5 Initial conditions	257
6.2.6 Change in dependent variable for homogeneous initial conditions	259
6.3 Methods of solution of the diffusion equation	262
6.3.1 Direct solution procedure	262
6.3.2 Trial function approach	264
6.3.3 Separation of variables method - Cartesian coordinates	266
6.3.4 Separation of variables method - plane polar coordinates	292
6.4 Some generalized results associated with the diffusion equation	301
6.4.1 Reduction to Helmholtz equation	301
6.4.2 Product solutions for the diffusion equation	302
6.4.3 Sturm-Liouville problems	306
6.5 Separation of variables method for spatially two-dimensional problems	312
6.5.1 Spatially two-dimensional problems - Cartesian coordinates	313
6.5.2 Spatially two-dimensional problems - plane polar coordinates	323
6.5.3 Product solutions and solutions for infinite domains	337
6.6 Uniqueness theorem	345
6.7 A maximum principle	349
6.8 PROBLEM SET 6	353
7. The wave equation	369
7.1 Wave motion in strings	371
7.1.1 Harmonic waves	374

7.1.2	d'Alembert's solution	378
7.1.3	Fourier analysis of the stretched string	385
7.1.4	Reflection and transmission at boundaries	386
7.1.5	Energy in a string	393
7.1.6	Forced motion of a semi-infinite string	397
7.1.7	Forced motion of an infinite string	402
7.2	Wave motion in stretched finite strings	415
7.2.1	Waves in a stretched finite string	415
7.2.2	Vibrations of a stretched finite string: trial function approach	418
7.2.3	Vibrations of a stretched finite string - variables separable solution	422
7.2.4	Vibrations of a stretched string: variable boundary conditions	431
7.2.5	Forced vibration of a stretched finite string	437
7.3	Wave motion in stretched strings: non-classical effects	446
7.3.1	Elastically supported string	446
7.3.2	Energy dissipation and damping in a stretched string	451
7.4	Waves and vibrations in stretched membranes	453
7.4.1	Equation of motion for a stretched membrane	454
7.4.2	Plane wave motion in a stretched infinite membrane	460
7.4.3	Free vibrations of a stretched membrane of infinite extent	463
7.4.4	Symmetric free vibrations of the stretched membrane	469
7.4.5	Green's function for the vibration of a stretched membrane	473
7.5	Vibrations of stretched finite membranes	475
7.5.1	Vibrations of a stretched square membrane	475
7.5.2	Free vibrations of a stretched rectangular membrane	482
7.5.3	Forced vibrations of a stretched rectangular membrane	489
7.5.4	Free vibrations of a stretched circular membrane	492
7.5.5	Hankel transform analysis of free vibrations of a stretched circular membrane	499
7.5.6	Hankel transform analysis of forced vibrations of a stretched circular membrane	501
7.5.7	Vibrations of a circular membrane-general formulation	505
7.6	Wave motion and vibrations in membranes: non-classical effects	511
7.7	Wave equation for problems in solid mechanics	512
7.7.1	Longitudinal wave motion in a slender elastic rod	513
7.7.2	Torsional waves in a slender circular elastic rod	522

7.8	Shallow water waves	525
7.9	Uniqueness theorem	531
7.10	PROBLEM SET 7	541
Bibliography		557
Index		587