

Table of Contents

Synopsis	1
 I Balance Equations	
1 Elements of Tensor Algebra and Analysis	9
1.1 Vectors and Second-Order Tensors	9
1.2 Symmetric Tensors	14
1.3 Skew and Orthogonal Tensors	19
1.4 Invertible Tensors	22
1.5 Bravais Lattices	24
1.6 Higher-Order Tensors	28
 2 Geometry and Kinematics of Continuous Bodies	 29
2.1 Processes with Singular Surfaces	29
2.2 Motion and Deformation	33
2.3 Compatibility of Deformations at the Interface	38
2.4 Rank 1 Connections	47
2.5 Twins	51
2.6 Appendix: Piecewise Smooth Objects	56
 3 Balance Equations	 61
3.1 Extensive Quantities: Fluxes	61
3.2 Extensive Quantities: Densities and Transport Theorems	65
3.3 Extensive Quantities: Balance Equations	67
3.4 Mass	70
3.5 Linear and Angular Momenta	72
3.6 Energy	74
3.7 Entropy	76
3.8 Appendix: The Gauss–Green Theorem	79

II Foundations

4	Material Bodies	89
4.1	State Space	89
4.2	Local State Functions; Material Bodies	91
5	The First Law of Thermodynamics	95
5.1	Work and Heat	95
5.2	Joule's Relation	96
5.3	Energy. The Equation of Balance of Energy	98
6	The Principle of Material Frame Indifference	101
6.1	Formulation	101
6.2	The Transformation Law for Work; Mass	104
6.3	Cauchy's Equations of Motion; Internal Energy	107
7	The Second Law of Thermodynamics	109
7.1	Empirical Temperature. The Heating Measure	109
7.2	Statements of the Second Law	115
7.3	Ideal Systems	116
7.4	The Collection of Bodies	121
7.5	The Absolute Temperature Scale. The Clausius Inequality	124
7.6	The Entropy. The Clausius–Duhem Inequality	127
7.7	Notes and Complements	132

III Constitutive Theory

8	Isotropic Functions	137
8.1	Isotropic Tensor-Valued Functions	137
8.2	Isotropic Scalar-Valued Functions	142
8.3	Objective Functions	143
8.4	Objective-Isotropic Tensor-Valued Functions	144
8.5	Objective-Isotropic Scalar-Valued Functions	147
9	Constitutive Equations	151
9.1	Response Functions	151
9.2	Consequences of the Clausius–Duhem Inequality	153
9.3	Frame Indifference	155
9.4	The Symmetry Group	157
9.5	Supply-Free Processes	161
10	The Equilibrium Response	167
10.1	The Legendre Transformation	167
10.2	Changes of Thermal Variables	170

10.3	The Eshelby Tensor. The Spatial Description	172
10.4	The Generalized Stress and Strain Measures	173
10.5	Isothermal Elastic Constants	174
10.6	The Thermal Coefficient of Stress	178
10.7	Adiabatic Elastic Constants	179
10.8	Specific and Latent Heats; Calorimetry	180
10.9	Approximate Equilibrium Response	182
11	The Equilibrium Response of Isotropic Bodies	185
11.1	Response Functions for Isotropic Solids	185
11.2	Isotropic States	188
11.3	Free Energies of Isotropic Solids	192
11.4	Response Functions of Fluids	193
12	The Dynamic Response	197
12.1	Linearization, Kinetic Coefficients	197
12.2	Linear Irreversible Thermodynamics. Onsager's Relations	199
12.3	Dissipation Potential	201
12.4	Relaxation Models. The Extended Linear Irreversible Thermodynamics	202
 IV Thermodynamic Equilibrium		
13	The Environment	209
13.1	States and Processes	209
13.2	Heating Environments	210
13.3	Loading Environments	213
13.4	The Total Canonical Free Energy	220
13.5	Homogeneous Null Lagrangians	221
13.6	General Null Lagrangians	224
13.7	The Form of the Potential Energy	226
14	Equilibrium States	229
14.1	Equilibrium States and Dissipation of Energy	229
14.2	Equilibrium States for Given Environments	230
14.3	Integral Functionals	233
14.4	Variational Conditions for Thermodynamic Equilibrium	236
14.5	Spatial Description. Standard, Inner, and Outer Variations	238
15	Extremum Principles	243
15.1	Liapunov Functions and Stability	243
15.2	The Extremum Principles	248
15.3	Relationships Among the Principles	250
15.4	Extremum Principles and Variations	251

16 Convexity	255
16.1 Convex Sets	255
16.2 Convex Functions	256
16.3 The Lower Convex Hull	260
16.4 The Fenchel Transformation	262
17 Constitutive Inequalities	267
17.1 Quasiconvexity	267
17.2 Quasiconvexity at the Boundary	272
17.3 Rank 1 Convexity and the Legendre–Hadamard Condition	274
17.4 Maxwell’s Relation	279
17.5 Convexity and Polyconvexity	284
17.6 The Exchange of the Actual and Reference Configurations	288
17.7 Constitutive Inequalities for Fluids	289
17.8 Quasiconvexity and Crystals	292
18 Convexity Conditions for Isotropic Functions	295
18.1 Symmetric Convex Functions and Sets	295
18.2 Isotropic Convex Functions and Sets	298
18.3 Objective-Isotropic Convex Functions	301
18.4 Invertibility of the Stress Relation	304
18.5 Isotropic Polyconvex Functions	307
18.6 The Second Differential of the Stored Energy	307
19 Thermostatistics of Fluids	311
19.1 Preview: The Energy Function	311
19.2 Rest States and Total Quantities	313
19.3 Extremum Principles for Fluids	315
19.4 The Equivalence and Consequences of the Extremum Principles	316
19.5 Strict Extremum Principles. The Phase Rule	321
19.6 The Gibbs Function	323
19.7 Strong Minima and Dynamical Stability of Equilibrium States	326
19.8 The Equilibrium of Fluids Under the Body Forces	327
20 A Local Approach to the Equilibrium of Solids	333
20.1 The Linearized Equations	333
20.2 Sobolev Spaces	338
20.3 The Second Variations and Extrema	340
20.4 Positivity of the Second Variation (Necessary Conditions)	343
20.5 Positivity of the Second Variation (Sufficient Conditions)	350
20.6 The Second Variation for Stressed Isotropic States	351
20.7 Stability and Bifurcation for a Column	360
20.8 Existence in Linearized Elasticity	363
20.9 Existence Via the Implicit Function Theorem	365

21 Direct Methods in Equilibrium Theory	369
21.1 Weak Convergence and Young Measures	370
21.2 Deformations from Sobolev Spaces	375
21.3 Weak Convergence of Determinant and Cofactor	379
21.4 States of Rubber-Like Bodies	381
21.5 Existence of Solutions to Extremum Problems for Rubber-Like Bodies	384
21.6 Minimum Energy in Crystals and Young Measure Minimizers	388
 V Dynamics	
22 Dynamical Thermoelastic and Adiabatic Theories	399
22.1 Equations of Dynamic Thermoelasticity	400
22.2 Extra Conditions for Evolving Phase Boundaries	402
22.3 Adiabatic and Isentropic Dynamics; Shock Waves	405
22.4 Equations in the Form of a First-Order System	409
 23 Waves in the Referential Description	411
23.1 The Characteristic Equation	411
23.2 Characteristic Fields. Genuine Nonlinearity	414
23.3 Plane, Surface, and Acceleration Waves	415
23.4 The Characteristic Equation and Material Symmetry	421
23.5 Centered Waves	424
23.6 Discontinuities	426
23.7 The Shock Set	429
23.8 The Shock Admissibility Criteria	434
23.9 The Riemann Problem	440
 24 Adiabatic Fluid Dynamics	443
24.1 The Equations of Fluid Dynamics	443
24.2 Shock Waves in Fluids	445
24.3 Hugoniot's Adiabats	447
24.4 The Equivalence of the Admissibility Criteria	452
24.5 Shock Layers in Fluids	453
 25 Dissipation of Energy in Solids	461
25.1 Review of Basic Equations	461
25.2 Liapunov Functions	465
25.3 Uniqueness	467
25.4 The Existence of the Linear Time Evolution	468
25.5 Asymptotic Stability	473
25.6 The Linearization About Nonequilibrium States	474

References 479

Subject Index 501