CONTENTS

I PHENOMENOLOGY OF FLUCTUATIONS: GINZBURG–LANDAU FORMALISM

1 Introduction 3

2 Fluctuation thermodynamics 7
2.1 Ginzburg–Landau theory 7
2.1.1 GL functional 7
2.1.2 Heat capacity jump 9
2.1.3 GL equations 9
2.2 Fluctuation contribution to heat capacity 12
2.2.1 Zero dimensionality: the exact solution 12
2.2.2 Arbitrary dimensionality: case $T \geq T_c$. 13
2.2.3 Arbitrary dimensionality: case $T < T_c$. 15
2.3 Fluctuation diamagnetism 16
2.3.1 Qualitative preliminaries 16
2.3.2 Zero-dimensional diamagnetic susceptibility 19
2.3.3 $2D$ magnetization 21
2.4 Layered superconductor in magnetic field 25
2.4.1 Lawrence–Doniach model 25
2.4.2 General formula for the fluctuation free energy in magnetic field 27
2.4.3 Fluctuation magnetization and its crossovers 28
2.4.4 Fluctuation heat capacity in magnetic field 31
2.5 Ginzburg–Levanyuk criterion 31
2.5.1 Definition of the Gi number from the heat capacity in zero magnetic field 31
2.5.2 Other definitions 33
2.5.3 Broadening of the critical region by a magnetic field 34
2.6 Scaling and the renormalization group 37
2.7 Effect of fluctuations on superfluid density and critical temperature 45
2.8 Fluctuations of magnetic field 50

3 Fluctuation transport 52
3.1 Time-dependent GL equation 52
3.2 General expression for paraconductivity 54
3.3 Paraconductivity of a layered superconductor 57
3.3.1 In-plane conductivity 58

3.3.2 Out-of plane conductivity 59
3.3.3 Analysis of the limiting cases 60
3.4 Hall paraconductivity 62
3.5 Magnetic field angular dependence of paraconductivity 65
3.6 Paraconductivity of nanotubes 69
3.6.1 Paraconductivity in zero magnetic field 71
3.6.2 Fluctuation magnetoconductivity 72
3.6.3 Discussion 73
3.7 Transport equation for fluctuation Cooper pairs 74

4 Fluctuations in vortex structures 78
4.1 Vortex lattice and magnetic flux resistivity 78
4.2 Collective pinning 80
4.2.1 Correlation length 81
4.2.2 Critical current 83
4.2.3 Collective pinning in other systems 84
4.2.4 Thermal depinning 84
4.3 Creep 85
4.4 Melting of the vortex lattice 89

II BASIC NOTIONS OF THE MICROSCOPIC THEORY

5 Microscopic derivation of the TDGL equation 95
5.1 Preliminaries 95
5.2 The Cooper channel of electron–electron interaction: the fluctuation propagator 96
5.3 Diagrammatic representation of fluctuation corrections 100
5.4 Superconductor with impurities 102
5.4.1 Accounting for electron scattering by impurities 102
5.4.2 Propagator for superconductor with impurities 104
5.5 Layered superconductor 107

6 Microscopic derivation of the GL functional 109
6.1 GL functional of a conventional superconductor 109
6.2 GL functional in the case of a nontrivial order parameter symmetry 114

7 Microscopic theory of fluctuation conductivity 118
7.1 Qualitative discussion of the different fluctuation contributions 118
7.2 The electromagnetic response operator 120
7.3 Fluctuation conductivity of a layered superconductor in the vicinity of T_c 122
7.3.1 AL contribution 122
7.3.2 Contributions from fluctuations of the DOS 126

7.3.3 MT contribution 131
7.3.4 Phase-breaking time τ_φ 138
7.3.5 Comparison with the experiment 142
7.4 Fluctuation conductivity in a.c. field 146
7.5 Fluctuation conductivity in ultra-clean superconductors 148
7.5.1 Preliminaries 148
7.5.2 Nonlocal fluctuation conductivity in quasi-ballistic regime 150
7.5.3 Discussion 156
7.6 Fluctuation conductivity far from T_c 157
7.6.1 Paraconductivity of clean superconductor for $T \gg T_c$ 157
7.6.2 Fluctuation conductivity of impure superconductor for $T \gg T_c$ 160
7.6.3 The effect of inter-electron interaction in Cooper channel on the conductivity of disordered metal 174
7.7 Nonlinear fluctuation effects 179

III MANIFESTATION OF FLUCTUATIONS IN OBSERVABLES

8 Fluctuations in magnetic field 185
8.1 Magnetoconductivity in the vicinity of transition 185
8.1.1 Self-consistent treatment of the critical temperature shift in magnetic field 190
8.2 Magnetoconductivity far from transition 192
8.3 Effect of fluctuations on the Hall conductivity 195
8.3.1 AL contribution to the Hall conductivity 195
8.3.2 Effect of topological singularity on fluctuation Hall effect 197
8.4 Fluctuation magnetic susceptibility far from transition 201
8.5 Fluctuations in magnetic fields above $H_{c2}(0)$ 203
8.5.1 Conductivity 203
8.5.2 Magnetization 207

9 DOS and tunneling 211
9.1 Density of states of superconductor in fluctuation regime 211
9.2 Fluctuation phenomena in tunnel junction above T_c 213
9.2.1 Preliminaries 213
9.2.2 The effect of fluctuations on the tunnel current above T_c 217
9.3 Fluctuation phenomena in N(S)-I-S junction 220
9.3.1 Quasiparticle current in N(S)-I-S junction 221

9.3.2 Effective boundary Hamiltonian 221
9.3.3 General formula for normal tunneling current 224
9.3.4 Andreev conductance 225
9.3.5 Pair-field susceptibility of a superconductor 226
9.3.6 Goldman's group experiments 227
9.4 Fluctuation tunneling anomaly in a superconductor above the paramagnetic limit 232
9.5 The magnetoresistance of a granular superconducting metal 238
9.5.1 Choice of the model 240
9.5.2 Cooperon and propagator for granular superconductor 241
9.5.3 Suppression of the conductivity due to DOS fluctuations 245
9.5.4 Zeeman splitting 248
9.5.5 Conclusions 250

10 Effect of fluctuations on thermoelectricity and heat transport 252
10.1 Equations of heat and electric transfer 252
10.2 Phenomenological definition of the heat current 253
10.3 Heat current operator for interacting electrons 254
10.4 Fluctuation thermoelectric power above the superconducting transition 256
10.4.1 Generalities 256
10.4.2 Effect of the DOS fluctuations 257
10.4.3 AL contribution to the thermoelectric power 259
10.4.4 Discussion 260
10.5 Thermal conductivity 261
10.6 Manifestation of superconducting fluctuations in the Nernst effect 263

11 Spin susceptibility and NMR 266
11.1 Preliminaries 266
11.2 Spin susceptibility 267
11.3 Relaxation rate 268
11.4 Discussion 270

IV FLUCTUATIONS IN NANOSTRUCTURES AND UNCONVENTIONAL SUPERCONDUCTING SYSTEMS

12 Fluctuations in nanograins, nanodrops and granular superconductors 273
12.1 Ultrasmall superconducting grains 273

12.2 Superconducting drops in system with quenched disorder 277
12.2.1 The optimal fluctuations in the vicinity of T_c 277
12.2.2 Formation of the superconducting drops in magnetic fields $H > \overline{H_{c2}}(0)$ 279
12.3 Exponential DOS tail in superconductor with quenched disorder 283
12.4 Josephson coupled superconducting grains and drops 285
12.5 Classical phase transition in granular superconductors 287
12.5.1 XY-model for granular superconductor 287
12.5.2 GL description of the granular superconductor 287
12.5.3 The broadening of superconducting transition by the quenched disorder 288
12.6 Quantum phase transition in granular superconductors 291
12.6.1 Coulomb suppression of superconductivity in the array of tunnel coupled granules 291
12.6.2 Superconducting grains in the normal metal matrix 292
12.6.3 Phase transition in disordered superconducting film in strong magnetic field 295

13 Fluctuations in Josephson junctions 299
13.1 General properties of a Josephson junction 299
13.1.1 Stationary Josephson effect 299
13.1.2 Nonstationary Josephson effect 303
13.2 Fluctuation broadening of the emission line 304
13.2.1 Thermal fluctuations of the voltage 304
13.2.2 Thermal fluctuations of the order parameter 308
13.3 Fluctuation suppression of the Josephson current below T_c 310
13.4 Josephson current decay due to the thermal phase fluctuations 313
13.5 Macroscopic quantum tunneling 317
13.5.1 Pair tunneling in the case of zero viscosity 319
13.5.2 Case of low viscosity: $Q \gg 1$ 321
13.5.3 Case of high viscosity: $Q \lesssim 1$ 322

14 Phase slip events 325
14.1 Classical phase-slip events 325
14.2 Quantum phase slip events in nanorings 327

15 Phase fluctuations in a $2D$ superconducting system 335
15.1 The crucial role of phase fluctuations in $2D$ systems 335
15.2 Exponential tail in the Josephson current close to T_c 336
15.3 Berezinskii-Kosterlitz-Thouless transition 338

15.4 Manifestation of vortex fluctuations above BKT transition 341

16 Fluctuations near superconductor-insulator transition 343
16.1 Quantum phase transition 343
16.2 $3D$ case 344
16.3 $2D$ superconductors 348
16.3.1 Preliminaries 348
16.3.2 Boson mechanism of T_c suppression 349
16.3.3 Fermion mechanism of T_c suppression 350

17 Role of fluctuations in high temperature superconductivity 352
17.1 Phase diagram 353
17.2 Resonating valence bond theory 354
17.2.1 Strongly correlated Fermi systems 354
17.2.2 Spin–charge separation 356
17.2.3 Hidden order phase 358
17.3 Bose-metal 359
17.3.1 Staggered flux state 360
17.3.2 Vortices localization 361
17.3.3 Bipolarons 362
17.4 BCS scenarios 362
17.4.1 Strong coupling BCS-like schemes 362
17.4.2 BEC–BCS scenario 363
17.5 Phenomenology 366
17.5.1 Marginal Fermi liquid 366
17.5.2 Bose phenomenology 367

A Relations between parameters of micro- and macroscopic theories 369
A.1 Coherence length 369
A.2 Other GL functional parameters 370

B Properties of the Euler gamma function and its logarithmic derivatives 372
B.1 Euler gamma function 372
B.2 Digamma function and its derivatives 372

C Integrals of the Lawrence–Doniach theory 375

References 377

Glossary 401

Index 409